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TREATISE ON VALVE-GEARS.

TREATISE

ON

VALVE-GEARS,

WITH SPECIAL CONSIDERATION

OF THE

LINK-MOTIONS OF LOCOMOTIVE ENGINES.

BY DR. GUSTAV ZEUNER,

PROFESSOR OF APPLIED MECHANICS AT THE CONFEDERATED POLYTECHNIKUM OF ZURICH.

TRANSLATED FROM THE FOURTH GERMAN EDITION.

BY PROFESSOR J. F. KLEIN,

LEHIGH UNIVERSITY, BETHLEHEM, PA.

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EDITOR'S PREFACE
TO THE REVISED ENGLISH EDITION.

In preparing this translation of the Fourth German Edition, the Editor's work consisted principally in making the retained portions of the Translation of the Third Edition somewhat less literal, in translating the additions to the First Part and the whole of the Second Part, and in correcting a few errors in the formulæ. The portions for which the Editor is alone responsible, are the tables on pp. 8 and 9, and the bracketed footnotes.

J. F. Klein.

Bethlehem, Pa., *July* 1884.

TRANSLATOR'S PREFACE

TO THE THIRD EDITION.

THE great circulation and the general acknowledgment which Professor Zeuner's Circle Diagram obtained in a few years amongst the German Engineers, and also the little knowledge of it which exists in England, have induced me, on the occasion of the publication of the Third Edition of Professor Zeuner's 'Treatise on Valve-Motions' to translate this edition into the English language. I now hand over to the public this Translation (which is as nearly as possible a literal one), and have only to add the desire, that the excellent work by Professor Zeuner may meet in England with the same success as it has in Germany, and that it may be as generally adopted.

MORITZ MÜLLER.

PREFACE

TO THE FOURTH GERMAN EDITION.

In preparing the present edition of "Valve-Gears," I have paid especial attention to the *second* part, which treats of double slide-valves (gears with independent cut-off). This part of the book has been entirely rewritten, and may, in a certain sense, be regarded as exhausting the subject. I have tried to solve the problem, not altogether easy, of discussing all the double valve-gears which are of interest and importance from the theoretical and practical standpoint, without too greatly increasing the size of the book; it was therefore necessary to present the subject in a short, precise fashion considering principally the important points, the subordinate points being merely indicated, or, in some cases, omitted. It is to be hoped that this condensed presentation will not render the book less clear or easy to understand. The principles employed in the theory of double slide-valves are not new; the principal proposition, which might be called *the parallelogram of eccentricities*, was deduced and applied in my first articles in the *Civil-Ingenieur*, and was also used in the later editions of this book in the discussion of *Meyer's* and *Polonceau's* valve-gears.

I thankfully acknowledge the assistance rendered me in preparing this edition by *Prof. L. Hauffe*, of the Technical School at Vienna, and *Prof. L. Kargl*, of the Polytechnic School at Zurich.

Gustav Zeuner.

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TREATISE

ON

VALVE-GEARS.

INTRODUCTION.

THE valve motion of steam engines has to perform the duty of admitting the steam directly from the boiler to the cylinder alternately before and behind the piston, and of releasing the used steam into the air in non-condensing engines, and into the condenser in engines with condensation.

The entrance and exhaustion of the steam take place through the two steam-ports, one of which leads to each end of the cylinder. Now, in the construction of the valve-gear, the problem is to open the entrances of these ports alternately by the application of peculiar arrangements at one time to the steam-pipe, and at the other time to a port which communicates either with the open air or with the condenser. These arrangements, which act directly in the manner stated, form the so-called *inside gear*, and consist of slide-valves, valves, or cocks.

Those parts of the steam engine which transfer motion from the engine itself to the above-mentioned arrangements, are comprised under the name "*outside gear.*"

They are divided principally into those gears in which this motion is given by eccentric discs (eccentrics), and those obtaining their motion from oscillating levers.

The slide-valve gears are those which are most applied and most

important, and as they alone lead to theoretical studies of the distribution of steam effected through them, an investigation into the seldom-applied valve and cock-gears has been entirely omitted in the following treatise. The slide-valve consists generally of a hollowed plate, which moves backwards and forwards over the entrances of the steam-ports; or if the slide-valve acts as a so-called expansion slide-valve, it only consists of a single plate, which may or may not have slots or ports for the admission of steam.

Slide-valve gears are divided into two kinds:—

(1) Slide-valve gears with one slide-valve; and
(2) Slide-valve gears with two slide-valves.

The slide-valve gears are always constructed in such manner as to admit the steam into the cylinder, not during the whole stroke of the piston, but only during a greater or smaller part of it; after the cutting off, the steam acts by its extension, or as it is called, by expansion. Now this earlier or later cut-off, or the greater or less expansion may be effected very well, as will be shown further on, by one single slide-valve; but there are certain limits for the expansion thus effected, and it very often happens that the distribution of steam effected by one slide-valve when cutting off soon proves to be disadvantageous. Slide-valve gears with two slide-valves are applied in such cases; besides the *distribution valve*, a second one is used, called the *expansion valve*, and which has only to regulate the admission and cutting off of the steam.

The motion of the slide-valve is produced, with very few exceptions, by means of eccentrics. For motions with one slide-valve there are employed *one* or *two* eccentrics; one eccentric is applied if the rotary motion of the engine takes place always in the same direction, while two eccentrics are generally applied if the motion of the engine has to be sometimes reversed. Motions of the latter kind, which are to be found with locomotive, marine, and winding engines, will be called in future "reversing motions."

The motions with two slide-valves have generally two or three eccentrics; in the first case, each of the two eccentrics moves one slide-valve; this arrangement is most applied to stationary engines, which

run always in the same direction. In the case of a reversing motion with two slide-valves, either three or two eccentrics are applied. If there are three eccentrics, one governs the motion of the expansion valve, and the two others the motion of the distribution valve; but in this case also two eccentrics only are very often applied.

The number of the eccentrics, however, cannot at all be taken as a starting point for the classification of the different systems of motions; should it appear desirable to classify the two sorts of motion—the motion with one valve and that with two valves—into subdivisions, the manner in which the expansion is governed would be the only base to start from, and both kinds of motion may then be divided as follows :—

(*a*) Motions with fixed or unvariable expansion.
(*b*) Motions with variable expansion.

In motions of the first kind the cutting-off of the steam by the slide-valve takes place always at the same point of the stroke of the piston, while in the case of motions with variable expansion the cut-off may be effected at any point of the stroke. The latter motions may again be distinguished according to whether an alteration of the expansion requires the stopping of the engine, or whether the alteration can be effected whilst the engine is running. But in the following pages no notice will be taken of the above-given classification, for a general view of the demonstration cannot be attained by it; and all motions examined in the following pages, with the exception of the motion with one valve and a fixed eccentric first considered, are those which allow a variable expansion, and in which the alteration of the expansion may be effected during the running of the engine.

FIRST PART.

VALVE-GEARS WITH ONE SLIDE-VALVE.

VALVE-GEARS WITH ONE SLIDE-VALVE.

SECTION I.

SIMPLE VALVE-GEAR WITH FIXED EXPANSION.

CHAPTER I.

DESCRIPTION OF THE SIMPLE VALVE-GEAR WITH D SLIDE-VALVE.

THE investigation of the subject of valve-gears may be introduced by a description of the principal parts of a horizontal steam-engine, such as is represented in Fig. 1, Pl. I.

The motion of the piston B, is transmitted through the piston-rod B A to the crosshead A, and from this through the connecting-rod A R to the pin R of the crank O R. The reciprocating motion of the piston is thus converted into the rotary motion of the crank.

The engine-shaft O carries, in addition to the crank, the *eccentric sheave* or *eccentric* D, which is a circular disk whose centre E lies outside of the shaft centre O; the line joining the points O and E makes a certain angle with the direction of the crank, and the length of this line is called the eccentricity. One end of the eccentric-rod F C encloses the eccentric with a ring called the *strap*, while the other end C is jointed to the valve-rod of the steam-valve S. As a consequence of the rotary motion of the engine-shaft, the eccentric fixed to this shaft will cause the end C of the valve-rod to

have a rectilinear, reciprocating motion, as if the motion proceeded from a crank of a length equal to the eccentricity O E of the eccentric.

The valve moves within the steam-chest K, which is connected with the boiler by the steam-pipe, and is, therefore, constantly filled with fresh steam. The cylinder is provided with three steam passages o_1, o and o_0, two of them o_1 and o, leading to the ends of the cylinder, and the third o_0 communicating either with the atmosphere or with the condenser. The entrances (or ports) to these passages from the steam-chest lie in a plane called the valve-seat, upon which the valve-face slides to and fro. The valve is hollowed on the inside and is called a D-valve, on account of its form. Now it is easy to see that fresh steam from the steam-chest will pass through the passage o_1 to that end of the cylinder which is on the *left* of the piston C, so long as the valve is sufficiently far to the *right* of its middle position. At the same time the steam to the right of the piston will escape from the cylinder through the passage o, and through the cavity in the valve to the exhaust-passage o_0, and from there to the atmosphere or condenser. On the other hand, when the valve is sufficiently far to the left of its middle position, the steam will enter through the right-hand passage o, and will exhaust through the left-hand passage o_1; in the latter case the piston usually travels backwards, that is from right to left, while in the former case it is moved by the steam from left to right; the two passages o_1 and o, therefore, regularly exchange their functions as entrance and exhaust passages.

The travel of the valve is under all circumstances so chosen that the inner cavity is always in communication with the exhaust passage o_0. This is effected by making the eccentricity small, or the width of the exhaust-passage o_0 large. Supposing these proportions to be correctly chosen, no further attention need be paid to this passage while investigating the distribution of steam in valve-gears, for the problem is then reduced to ascertaining how far the two steam-ports of o_1 and o are *open* for the admission or discharge of steam when the piston stands in any position.

But it may be shown that both of these port openings can be easily determined, when, for the given piston position, we know how

far the centre of the valve is from the centre of oscillation of the valve, i. e. when we know the movement of the valve.*

The piston positions are determined by the distances of the piston from the end of the stroke, that end of the stroke being chosen as a point of reference from which the piston is receding at the instant. These distances we will call *piston positions*, and will distinguish between the piston positions for the forward and for the return stroke. The whole problem to be solved in connection with both simple and complex valve-gears, may be briefly stated to be: *the analytical or graphical determination of the relation between the piston position and the valve position (movement).*

But the whole problem may be divided into two parts, and the resulting method of treatment is the one which is exclusively followed in the present work.

A certain position of the piston always corresponds to a perfectly determinate position of the crank. If the piston B is at the left end B_1 of its stroke (Fig. 1, Pl. I.), the crank-pin R will be at R_1, i. e. at the first *dead point*, and if the piston has traversed the distance B_1 B, the crank will have turned through the angle R_1 O R; consequently there is a definite relation between the piston position and the crank angle, and this relation can be determined independently of the arrangement of the valve-gear; its determination constitutes the *first* portion of the problem.

On the other hand, for the same crank angle the distance of the valve from its centre of oscillation will vary with the dimensions and arrangement of the ports of the valve-gear, there is, therefore, a definite relation between the valve position and the crank angle, and the establishing of this relation constitutes the *second* part of the problem.

Expressed analytically, we can say that both the piston position

* [In its most common acceptation, *travel* is synonymous with *stroke*, and is therefore not a suitable word for designating the variable distance of the centre of the valve from its centre of motion. In the present edition the term movement will be employed in place of travel (used in preceding edition) to designate this variable distance. Another term frequently occurring in valve-gear discussions is *throw*, by some writers it is treated as equivalent to stroke, by others to half-stroke, or to the radial reach of a crank, eccentric or cam. We shall use it in the latter sense. Throw is usually applied to rotating pieces, while *travel* is used in connection with sliding pieces, as pistons or valves.—Ed.]

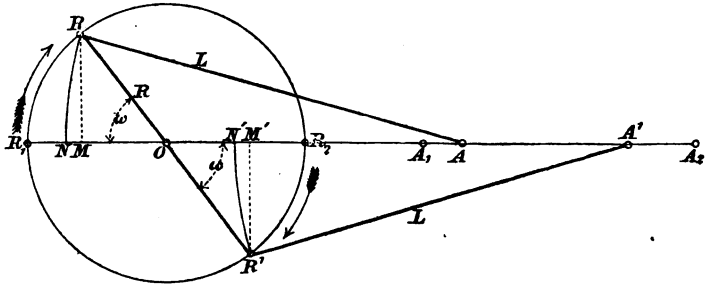
and the valve position can be represented as functions of the crank angle, and combining both equations and eliminating the crank angle, we get the direct relation between the piston and valve positions.

Relation between Piston Position and Crank Angle.

If the piston B (Fig. 1, Pl. I.) is at either end B_1 or B_2 of the stroke, the crank-pin will be at one of the dead points R_1 or R_2, and the crosshead will be at one of the ends A_1 or A_2 of its stroke. If the crank has turned in the direction of the arrow through the angle $R_1 O R$, which we will henceforth designate by ω, the piston will have traversed the distance $B_1 B$, which distance we will designate by s; this same distance is also traversed by the crosshead A, and we may, therefore, in the following investigation, consider only the motion of the latter and may then easily determine the movement $A_1 A = s$ as follows.

If we suppose the connecting-rod to occupy the position A R, and to be detached from the crank-pin R, and then turned about the fixed point A till the direction of the rod coincides with that of the stroke, then the end R, Fig. 1, will describe the arc R N, with the

Fig. 1.



length L of the connecting-rod A R as a radius; R represents the length O R of the crank.

We now see that the distance $R_1 N$ represents the distance s through which the piston has moved, and this distance is easily

represented by a formula. If we drop from the pin R the perpendicular R M on to the direction of the stroke, and for the present designate the rise M N of the arc R M by x, we have

$$s = R_1 N = O R_1 - O M - M N,$$

or,

$$s = R (1 - \cos \omega) - x.$$

This formula applies to the *forward* stroke of the piston, for the motion of the crank through the *first* (upper) semicircle.

On the other hand, for the *return* stroke of the piston, and for the motion of the crank through the *second* semicircle, we estimate the crank angle ω from the second dead point R_2, and estimate the distance $A_2 A'$, through which the piston has moved, from the end A_2 of its stroke. If we again describe, with the connecting-rod $A' R' = L$, the arc $R' N'$, and drop the perpendicular $R' N'$ on to the direction of the stroke, we shall find for the distance through which the piston has moved on the *return* stroke

$$s = R_2 N' = O R_2 - O M' + M' N',$$

or,

$$s = R (1 - \cos \omega) + x,$$

where x also represents the rise M N of the arc.

If we unite these two formulas for s, we get the general equation

$$s = R (1 - \cos \omega) \mp x, \qquad (1)$$

in which the upper sign refers to the forward stroke, and the lower sign to the return stroke of the piston.

To determine the value x, describe with the connecting-rod as a radius a complete circle with A or A′ as a centre, this circle will have a diameter 2 L, and the perpendicular $R M = R' M' = R \sin \omega$ will be the mean proportional between the segments x and $2 L - x$ of the diameter; we therefore have the equation

$$x (2 L - x) = R^2 \sin^2 \omega, \qquad (2)$$

and from this we get

$$x = L - \sqrt{L^2 - R^2 \sin^2 \omega},$$

and then from equation (1) the distance of the piston from the end of the stroke,

$$s = R(1 - \cos \omega) \mp L \pm \sqrt{L^2 - R^2 \sin^2 \omega}. \qquad (3)$$

But this formula is not convenient for practical use, and we may therefore employ approximate formulas. The value of x is always very small in comparison with 2 L, and it may, therefore, be omitted in the parenthesis occurring in the left member of equation (2). We then have, with sufficient accuracy for all practical purposes,

$$x = \frac{R^2 \sin^2 \omega}{2\,L},$$

and substituting this in equation (1), we get this first approximate formula for the distance of the piston from the end of the stroke,

$$s = R(1 - \cos \omega) \mp \frac{R^2}{2\,L} \sin^2 \omega. \qquad (4)$$

These equations show the remarkable result, that the distance through which the piston has moved from the end of its stroke is *different* for the forward and return stroke, even when the angles through which the crank has passed from the dead points are equal, moreover the formulas show that this difference is greater the shorter the connecting-rod.

When the connecting-rod is comparatively short, this difference causes inequalities in the distribution of the steam by the valve-gears, which are never wholly removed in the ordinary arrangement of the gears; there is only one way of reducing this difference to a practical minimum, and that is to make the connecting-rod of steam-engines as long as possible; we therefore rarely find the ratio L : R less than 5.

When the connecting-rod is sufficiently long, the value x in equation (1) may be neglected, and we then get the second approxi-

mate formula for the distance of the piston from the end of its stroke, namely,

$$s = R\,(1 - \cos\omega).^{*} \qquad (5)$$

When equation (5) is employed, R_1 M and R_2 M′, Fig. 1, represent the distances of the piston from the ends of the stroke when the crank angle is ω, and for equal crank angles these distances are equal on the forward and return stroke. These distances may be determined graphically for any position of the crank by dropping a perpendicular from the pin R on to the direction of the stroke; the distance of the point M of the perpendicular from the ends of the diameter R_1 R_2 gives approximately the distance of the piston from the end of the stroke at the instant in question. This method will be employed in what follows. If an exact determination of the piston positions is of importance to the designer, he may easily obtain them, graphically or analytically, from what has been established above.† When this is done in connection with the following investigations, we can at once ascertain the hurtful influence on the distribution of the steam of too short a connecting-rod, and we can also see how a slight variation of the proportions of the valve-gear will remedy the defect. Moreover, it is worth noticing that the double sign occurring in equations (1), (3), and (4) disappears when the crank angle ω is not estimated separately for each semicircle, but from the first dead point R through the whole circle, and when the piston positions are

* [The difference between this approximate formula and the exact one given by equation (1) is the quantity x, and its maximum value $= L - \sqrt{L^2 - R^2}$. Consequently the maximum error made by using equation (5) becomes, for

$\frac{L}{R} = 2$	2·5	3	4	5
0·268 R	0·209 R	0·172 R	0·127 R	0·101 R
6	7	8	9	10
0·084 R	0·072 R	0·063 R	0·056 R	0·050 R.—Ed.]

† [The piston positions for various crank angles may also be found *exactly* from the two tables given on the following pages. To obtain the desired piston positions it is only necessary to multiply the length of stroke by the tabular quantity corresponding to the crank angles assumed.—Ed.]

DISTANCE OF PISTON FROM BEGINNING OF STROKE WHEN STROKE IS *towards* CRANK-SHAFT.

Crank Angles.	Connecting-rod ÷ Crank =								Crank Angles.	Connecting-rod ÷ Crank =							
	4	4·5	5	5·5	6	7	8	∞		4	4·5	5	5·5	6	7	8	∞
2	·0004	·0004	·0004	·0004	·0004	·0003	·0003	·0003	92	·5809	·5737	·5679	·5632	·5594	·5533	·5487	·5174
4	·0015	·0015	·0015	·0014	·0014	·0014	·0014	·0012	94	·5981	·5909	·5851	·5805	·5766	·5706	·5661	·5349
6	·0034	·0033	·0033	·0032	·0032	·0031	·0030	·0027	96	·6151	·6079	·6022	·5976	·5938	·5878	·5833	·5523
8	·0061	·0060	·0058	·0057	·0057	·0056	·0055	·0049	98	·6318	·6247	·6191	·6145	·6107	·6048	·6004	·5696
10	·0095	·0092	·0091	·0089	·0089	·0087	·0085	·0076	100	·6484	·6414	·6358	·6313	·6275	·6216	·6172	·5868
12	·0136	·0133	·0131	·0129	·0127	·0126	·0124	·0109	102	·6647	·6578	·6523	·6478	·6441	·6384	·6340	·6040
14	·0185	·0181	·0178	·0175	·0173	·0170	·0167	·0149	104	·6807	·6739	·6685	·6641	·6605	·6548	·6505	·6210
16	·0241	·0236	·0232	·0228	·0225	·0221	·0218	·0194	106	·6964	·6897	·6845	·6802	·6766	·6710	·6668	·6378
18	·0304	·0298	·0293	·0290	·0285	·0279	·0275	·0245	108	·7119	·7053	·7002	·6959	·6924	·6870	·6829	·6545
20	·0375	·0367	·0360	·0355	·0350	·0344	·0339	·0302	110	·7270	·7206	·7156	·7115	·7080	·7027	·6987	·6710
22	·0452	·0442	·0434	·0428	·0423	·0414	·0408	·0364	112	·7418	·7356	·7307	·7267	·7233	·7181	·7143	·6873
24	·0536	·0524	·0515	·0507	·0501	·0491	·0484	·0432	114	·7562	·7502	·7455	·7416	·7383	·7334	·7296	·7034
26	·0627	·0613	·0602	·0594	·0586	·0575	·0566	·0506	116	·7703	·7645	·7599	·7561	·7530	·7482	·7445	·7192
28	·0724	·0708	·0696	·0686	·0677	·0664	·0654	·0585	118	·7841	·7785	·7740	·7704	·7674	·7626	·7591	·7347
30	·0827	·0809	·0796	·0784	·0774	·0759	·0748	·0670	120	·7974	·7921	·7878	·7843	·7814	·7769	·7735	·7500
32	·0936	·0916	·0901	·0888	·0877	·0860	·0848	·0760	122	·8104	·8053	·8012	·7979	·7951	·7908	·7875	·7650
34	·1051	·1029	·1012	·0997	·0985	·0967	·0953	·0855	124	·8230	·8181	·8142	·8105	·8084	·8042	·8011	·7796
36	·1172	·1147	·1128	·1112	·1099	·1079	·1063	·0955	126	·8352	·8305	·8268	·8238	·8213	·8173	·8144	·7939
38	·1298	·1271	·1250	·1233	·1218	·1196	·1179	·1060	128	·8470	·8426	·8391	·8362	·8338	·8300	·8273	·8078
40	·1430	·1400	·1377	·1358	·1342	·1318	·1297	·1170	130	·8584	·8542	·8509	·8482	·8459	·8424	·8398	·8214
42	·1566	·1534	·1509	·1489	·1471	·1444	·1424	·1284	132	·8694	·8655	·8623	·8598	·8577	·8544	·8519	·8346
44	·1707	·1673	·1646	·1623	·1605	·1576	·1557	·1403	134	·8799	·8763	·8733	·8710	·8690	·8658	·8635	·8473
46	·1853	·1816	·1787	·1763	·1743	·1712	·1689	·1527	136	·8901	·8866	·8839	·8817	·8798	·8770	·8751	·8597
48	·2003	·1963	·1932	·1906	·1885	·1852	·1827	·1654	138	·8998	·8966	·8941	·8920	·8903	·8876	·8856	·8716
50	·2156	·2115	·2081	·2054	·2032	·1996	·1970	·1786	140	·9090	·9061	·9038	·9019	·9003	·8978	·8959	·8830
52	·2314	·2270	·2234	·2206	·2182	·2144	·2117	·1922	142	·9178	·9152	·9130	·9113	·9098	·9076	·9059	·8940
54	·2474	·2428	·2391	·2360	·2335	·2295	·2266	·2061	144	·9262	·9238	·9218	·9203	·9189	·9169	·9153	·9045
56	·2638	·2589	·2550	·2518	·2492	·2450	·2419	·2204	146	·9342	·9320	·9302	·9288	·9276	·9257	·9243	·9145
58	·2805	·2754	·2713	·2679	·2652	·2608	·2575	·2350	148	·9417	·9397	·9381	·9368	·9358	·9340	·9328	·9240
60	·2974	·2920	·2878	·2843	·2814	·2769	·2735	·2500	150	·9487	·9469	·9455	·9444	·9435	·9419	·9408	·9330
62	·3146	·3090	·3046	·3009	·2979	·2932	·2897	·2653	152	·9553	·9538	·9525	·9515	·9507	·9494	·9484	·9415
64	·3320	·3262	·3215	·3178	·3147	·3098	·3061	·2808	154	·9617	·9601	·9590	·9581	·9574	·9563	·9554	·9494
66	·3495	·3435	·3387	·3348	·3316	·3266	·3228	·2966	156	·9671	·9660	·9651	·9643	·9637	·9627	·9620	·9568
68	·3672	·3610	·3561	·3520	·3487	·3435	·3397	·3127	158	·9724	·9714	·9706	·9700	·9694	·9686	·9678	·9634
70	·3850	·3786	·3735	·3694	·3660	·3607	·3567	·3290	160	·9772	·9763	·9757	·9751	·9747	·9740	·9735	·9698
72	·4028	·3963	·3911	·3869	·3834	·3780	·3739	·3455	162	·9815	·9808	·9803	·9801	·9795	·9789	·9785	·9755
74	·4208	·4141	·4088	·4045	·4009	·3954	·3912	·3622	164	·9854	·9848	·9844	·9841	·9838	·9833	·9830	·9806
76	·4388	·4319	·4266	·4222	·4185	·4128	·4085	·3790	166	·9888	·9884	·9881	·9878	·9878	·9872	·9869	·9851
78	·4568	·4498	·4444	·4399	·4362	·4304	·4260	·3960	168	·9918	·9913	·9912	·9910	·9909	·9907	·9905	·9891
80	·4747	·4677	·4622	·4576	·4539	·4480	·4436	·4132	170	·9943	·9941	·9939	·9938	·9937	·9935	·9933	·9924
82	·4927	·4857	·4799	·4754	·4716	·4656	·4612	·4304	172	·9963	·9962	·9961	·9960	·9959	·9958	·9957	·9951
84	·5105	·5034	·4977	·4930	·4892	·4832	·4787	·4477	174	·9979	·9979	·9978	·9978	·9977	·9977	·9976	·9973
86	·5283	·5211	·5154	·5107	·5069	·5008	·4963	·4651	176	·9990	·9990	·9990	·9990	·9990	·9990	·9990	·9988
88	·5460	·5388	·5330	·5283	·5245	·5185	·5139	·4826	178	·9998	·9997	·9997	·9997	·9997	·9997	·9997	·9997
90	·5635	·5563	·5505	·5453	·5420	·5359	·5314	·5000	180	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000

DISTANCE OF PISTON FROM BEGINNING OF STROKE WHEN STROKE IS *away from* CRANK-SHAFT.

Crank Angles.	Connecting-rod ÷ Crank =								Crank Angles.	Connecting-rod ÷ Crank =							
	4	4·5	5	5·5	6	7	8	∞		4	4·5	5	5·5	6	7	8	∞
2	·0002	·0003	·0003	·0003	·0003	·0003	·0003	·0003	92	·4540	·4612	·4670	·4717	·4755	·4815	·4861	·5174
4	·0010	·0010	·0010	·0010	·0010	·0010	·0010	·0012	94	·4717	·4789	·4846	·4893	·4931	·4992	·5037	·5349
6	·0021	·0021	·0022	·0022	·0023	·0023	·0024	·0027	96	·4895	·4966	·5023	·5070	·5108	·5168	·5213	·5523
8	·0036	·0038	·0039	·0040	·0041	·0042	·0043	·0049	98	·5073	·5143	·5201	·5246	·5284	·5344	·5388	·5696
10	·0057	·0059	·0061	·0062	·0063	·0065	·0067	·0076	100	·5253	·5323	·5378	·5424	·5461	·5520	·5564	·5868
12	·0082	·0087	·0088	·0090	·0091	·0093	·0095	·0109	102	·5432	·5502	·5556	·5601	·5638	·5696	·5740	·6140
14	·0112	·0116	·0119	·0122	·0122	·0128	·0131	·0149	104	·5612	·5681	·5734	·5778	·5815	·5872	·5915	·6210
16	·0146	·0152	·0156	·0159	·0162	·0167	·0170	·0194	106	·5792	·5859	·5912	·5955	·5991	·6046	·6088	·6378
18	·0185	·0192	·0197	·0199	·0205	·0211	·0215	·0245	108	·5972	·6037	·6089	·6131	·6166	·6220	·6261	·6545
20	·0228	·0237	·0243	·0248	·0253	·0260	·0265	·0302	110	·6150	·6214	·6265	·6306	·6340	·6393	·6433	·6710
22	·0276	·0286	·0294	·0300	·0306	·0314	·0322	·0366	112	·6328	·6390	·6439	·6480	·6513	·6565	·6603	·6873
24	·0329	·0340	·0349	·0357	·0363	·0373	·0380	·0432	114	·6505	·6565	·6613	·6652	·6684	·6734	·6772	·7034
26	·0385	·0399	·0410	·0419	·0426	·0437	·0446	·0506	116	·6680	·6738	·6785	·6822	·6853	·6902	·6939	·7192
28	·0447	·0462	·0475	·0485	·0493	·0506	·0516	·0585	118	·6854	·6910	·6954	·6991	·7021	·7068	·7103	·7347
30	·0513	·0531	·0545	·0556	·0565	·0581	·0592	·0670	120	·7026	·7080	·7122	·7157	·7186	·7231	·7265	·7500
32	·0583	·0603	·0619	·0632	·0642	·0660	·0671	·0760	122	·7195	·7246	·7287	·7321	·7348	·7392	·7425	·7650
34	·0658	·0680	·0698	·0712	·0724	·0743	·0757	·0855	124	·7362	·7411	·7450	·7482	·7508	·7550	·7581	·7796
36	·0738	·0762	·0782	·0797	·0811	·0831	·0847	·0955	126	·7526	·7572	·7609	·7640	·7665	·7705	·7734	·7939
38	·0822	·0848	·0870	·0887	·0902	·0924	·0941	·1060	128	·7686	·7730	·7766	·7794	·7818	·7856	·7883	·8078
40	·0910	·0939	·0962	·0981	·0997	·1022	·1041	·1170	130	·7844	·7885	·7919	·7946	·7968	·8004	·8030	·8214
42	·1002	·1034	·1059	·1080	·1097	·1124	·1144	·1284	132	·7997	·8037	·8068	·8094	·8115	·8148	·8173	·8346
44	·1099	·1134	·1161	·1183	·1202	·1230	·1249	·1403	134	·8147	·8184	·8213	·8237	·8257	·8288	·8311	·8473
46	·1201	·1237	·1267	·1290	·1310	·1342	·1365	·1527	136	·8293	·8327	·8354	·8377	·8395	·8424	·8443	·8597
48	·1306	·1345	·1377	·1402	·1423	·1456	·1481	·1654	138	·8434	·8466	·8491	·8511	·8529	·8556	·8576	·8716
50	·1416	·1458	·1491	·1518	·1541	·1576	·1602	·1786	140	·8570	·8600	·8623	·8642	·8658	·8682	·8703	·8830
52	·1530	·1574	·1609	·1638	·1662	·1700	·1727	·1922	142	·8702	·8729	·8750	·8767	·8782	·8804	·8821	·8940
54	·1648	·1695	·1732	·1762	·1787	·1827	·1856	·2061	144	·8828	·8853	·8872	·8888	·8901	·8921	·8937	·9045
56	·1769	·1819	·1858	·1894	·1916	·1958	·1989	·2204	146	·8949	·8971	·8988	·9003	·9015	·9033	·9047	·9145
58	·1896	·1947	·1988	·2021	·2049	·2092	·2125	·2350	148	·9064	·9084	·9099	·9112	·9123	·9140	·9152	·9240
60	·2026	·2079	·2122	·2157	·2186	·2231	·2265	·2500	150	·9173	·9191	·9204	·9216	·9226	·9241	·9252	·9330
62	·2159	·2215	·2260	·2296	·2326	·2374	·2409	·2653	152	·9276	·9292	·9304	·9314	·9323	·9336	·9346	·9415
64	·2297	·2355	·2401	·2439	·2470	·2518	·2555	·2808	154	·9373	·9387	·9398	·9406	·9414	·9425	·9434	·9494
66	·2438	·2498	·2545	·2584	·2617	·2666	·2704	·2966	156	·9464	·9476	·9485	·9493	·9499	·9509	·9516	·9568
68	·2582	·2644	·2693	·2733	·2767	·2819	·2857	·3127	158	·9548	·9558	·9566	·9572	·9577	·9586	·9592	·9636
70	·2730	·2794	·2844	·2885	·2920	·2973	·3013	·3290	160	·9625	·9633	·9640	·9645	·9650	·9656	·9661	·9698
72	·2881	·2947	·2998	·3041	·3076	·3130	·3171	·3455	162	·9696	·9702	·9707	·9710	·9715	·9721	·9725	·9755
74	·3036	·3103	·3155	·3198	·3234	·3290	·3332	·3622	164	·9759	·9764	·9767	·9772	·9775	·9779	·9782	·9806
76	·3193	·3261	·3315	·3359	·3395	·3452	·3495	·3790	166	·9815	·9819	·9822	·9825	·9827	·9830	·9833	·9851
78	·3353	·3422	·3477	·3522	·3559	·3616	·3660	·3960	168	·9864	·9867	·9869	·9871	·9873	·9874	·9876	·9891
80	·3516	·3586	·3642	·3687	·3725	·3784	·3828	·4132	170	·9905	·9908	·9909	·9911	·9911	·9913	·9915	·9924
82	·3682	·3753	·3809	·3855	·3893	·3952	·3996	·4304	172	·9939	·9940	·9942	·9943	·9943	·9944	·9945	·9951
84	·3849	·3921	·3978	·4024	·4062	·4122	·4167	·4477	174	·9966	·9967	·9967	·9968	·9968	·9969	·9970	·9973
86	·4019	·4091	·4149	·4195	·4234	·4294	·4339	·4651	176	·9985	·9985	·9985	·9986	·9986	·9986	·9986	·9988
88	·4191	·4263	·4321	·4368	·4406	·4467	·4513	·4826	178	·9996	·9996	·9996	·9996	·9996	·9997	·9997	·9997
90	·4365	·4437	·4495	·4547	·4580	·4641	·4686	·5000	180	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000

not estimated from the ends of the stroke, *but from the middle of the stroke;* if s_m is the distance of the piston from the middle, i.e. $s_m = R - s$, we have for *all* crank angles

$$s_m = R \cos \omega + x,$$

in which equation we can substitute the exact value for x given by equation (2) or its approximate value. But it is not customary nor convenient to determine the piston positions in this manner, for, in discussing the admission, expansion, and compression of the steam in the cylinder of the engine, it is the distance of the piston from the end of the stroke that must be taken into account.

Relation between Valve Position and Crank Angle.

Theory of the Valve Motion.

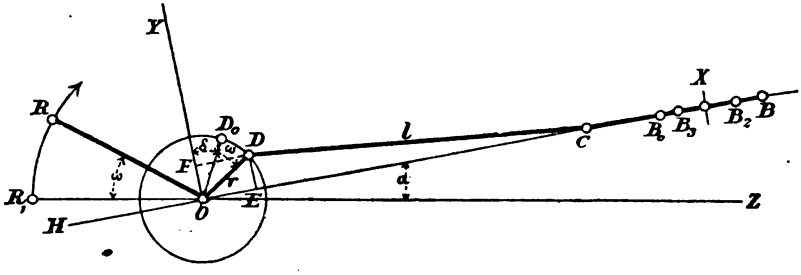
The determination of the relation between the valve movement, i.e. the distance of the valve from its middle position, and the crank angle, constitutes the principal problem in the theory of valve-gears; before proceeding to establish the fundamental formula for the simple valve-gear, a few remarks are necessary. The engine arrangement shown in Fig. 1, Pl. I., is assumed as the basis of the discussion.

If the axis C S of the valve-rod be prolonged, its direction will pass through the centre O of the shaft, and the straight line H O C becomes an important line of reference. In the first place, it gives the direction in which *the end* C *of the eccentric-rod* moves with a reciprocating motion, and this direction *must always* pass through the centre O of the shaft. We call this the *valve direction;* in the arrangement shown in Fig. 1, Pl. I., this direction coincides with that of the valve, and the valve-seat is parallel to it; but this is by no means always the case, for the motion of the end C of the eccentric-rod F C is often transmitted to the end of the valve-rod by a one- or two-armed lever. In such a case the end C swings in an *arc* whose *chord* coincides with the line O C. We will recur to the

last-mentioned case later, and will here assume that the motion is transmitted *directly* to the valve, as shown in the figure; the case of indirect transmission can then be easily reduced to that of direct transmission.

Let O Z, Fig. 2, be the direction of the axis of the cylinder, and O C the valve direction; let a be the angle which these two directions make with each other; let O R be the crank, O D $= r$ the eccentricity, and D C $= l$ the length of the eccentric-rod; let B

FIG. 2.



represent the centre of the valve, and C B $= l_1$ be the length of the valve-rod measured to the centre of the valve. Now, if we suppose the pin R to be brought on to the dead point, i. e. the crank O R turned back through the angle ω till the piston stands at the end of its stroke, then the eccentricity O D will move to O D_0, and in *this* position will make *with a perpendicular* O Y *to the valve direction* O C an angle Y O $D_0 = \delta$, which plays an important part in the theory of valve motions; this angle is called the *angle of advance.* In this connection it is well to note that the angle δ is entirely independent of the direction of the axis O Z of the cylinder relatively to the *valve direction* O C, and that the latter angle a is only brought in when it is a question of determining the position of the eccentricity relatively to the crank, i. e. when the angle R_1 O D_0 = R O D is to be determined; in the present case it is evident from the figure that this angle amounts to $(90° + \delta - a)$; but in accordance with the above definition of the angle of advance, the angle R O D and the angle a do not enter into the investigation of the valve motion, nor into that of the distribution of steam resulting from this

motion.* Now, in order to establish the fundamental equation, we will assume that the crank has moved from the dead point through the angle ω, and consequently that eccentricity has moved from O D_0 to O D, and that the centre of the valve is at B; the distance O B of the latter from the *centre* O *of the shaft* may then be determined as follows:—

If we drop the perpendicular D E on to the valve direction, and draw D F parallel to O E, we have from Fig. 2,

$$OB = OE - EC + CB = DF + \sqrt{\overline{DC}^2 - \overline{OF}^2} + CB,$$

or,

$$OB = r \sin(\omega + \delta) + \sqrt{l^2 - r^2 \cos^2(\omega + \delta)} + l_1.$$

This radical can also be written

$$\sqrt{l^2 - r^2 \cos^2(\omega + \delta)} = l \sqrt{1 - \frac{r^2}{l^2} \cos^2(\omega + \delta)},$$

and the radical in the right member may be expanded according to the binomial theorem, and only the first two terms of the series need be retained, because the length l of the eccentric-rod is always very great relatively to the eccentricity r. We therefore have

$$\sqrt{l^2 - r^2 \cos^2(\omega + \delta)} = l \left(1 - \frac{r^2}{2\,l^2} \cos^2(\omega + \delta)\right),$$

and substituting this approximate value in the above equation for O B we get, with sufficient accuracy for all cases,

$$OB = r \sin(\omega + \delta) + l + l_1 - \frac{r^2 \cos^2(\omega + \delta)}{2\,l}.$$

But by the nature of the arrangement, the centre B of the slide-valve must move symmetrically backwards and forwards on either

* [According to Rankine, angle of advance = phase of eccentric − phase of crank − 90°. When crank is at its first dead point, the angular advance = phase of eccentric − 90°, which agrees with the definition given above. The phase or angle of the crank is estimated from its first dead point and the phase of the eccentric arm from its first dead point; usually these two reference lines (the "dead points" of crank and eccentric arm) coincide, but cases occur in which they do not coincide, as in Fig. 2, where the direction of the valve-stroke is inclined to axis of cylinder.—Ed.]

side of a fixed point X in such a manner that the centre of the slide-valve, at the instants when the crank is passing the points ω and $180^\circ - \omega$, is at equal distances, but on opposite sides of the point X. Many designers adjust the slide-valve by bringing the crank alternately on its dead points, that is by taking $\omega = 0$ and $\omega = 180^\circ$, and then marking the two corresponding positions B_2 and B_3 of the centre of the slide-valve and fixing the centre X of the distance $B_2\ B_3$; this centre X is the one from which the movement of the valve will be estimated, and must fall exactly in the middle of the exhaust-port. This method of proceeding is, as we shall show presently, also theoretically the most correct one, and we shall apply it in order to calculate the position of X or the distance O X. If in the last formula for obtaining O B, we for once make $\omega = 0$, we get

$$O\,B_2 = r \sin \delta + l_1 + l - \frac{r^2 \cos^2 \delta}{2\,l};$$

whilst if we make $\omega = 180^\circ$, we get

$$O\,B_3 = -\,r \sin \delta + l_1 + l - \frac{r^2 \cos^2 \delta}{2\,l}.$$

The mean of these two values gives the distance of the centre of motion * X from the centre of the axle,

$$O\,X = l + l_1 - \frac{r^2 \cos^2 \delta}{2\,l}. \qquad (6)$$

But the whole object of an investigation of the motion of the slide-valve is to enable us *to fix the distance of the slide-valve centre* B *from the centre of motion* X, *corresponding to any angular movement* ω *of the axle*, or as we may say, *to fix the movement of the valve for any crank angle* ω.

* [The centre of motion X should not be understood as centre of stroke of the valve, for this reference point X is at the distance $\frac{r^2 \cos^2 \delta}{2\,l}$ from the centre of stroke. When the length of the eccentric-rod is infinite (the case usually assumed in the formulas) the two centres coincide.—Ed.]

According to Fig. 2, the movement of the valve is $BX = OB - OX$; calling this ξ and using for OB and OX the above given values, we get, after a few reductions,

$$(I^a) \qquad \xi = r \sin(\omega + \delta) + \frac{r^2}{2l}[\cos^2\delta - \cos^2(\omega + \delta)].$$

A transformation* of the quantities between the brackets gives

$$\xi = r \sin(\omega + \delta) + \frac{r^2}{2l} \sin(2\delta + \omega) \sin\omega,$$

or, by expanding the first term,

$$(I^b) \qquad \xi = r \sin\delta \cos\omega + r \cos\delta \sin\omega + \frac{r^2}{2l} \sin(2\delta + \omega) \sin\omega.$$

If we make

$$r \sin\delta = A \tag{7}$$

$$r \cos\delta = B, \tag{8}$$

A and B being constant quantities for any particular slide-valve gear, and

$$\frac{r^2}{2l} \sin(2\delta + \omega) \sin\omega = F,$$

we get

$$(I^c) \qquad \xi = A \cos\omega + B \sin\omega + F.$$

The last quantity F is always very small, because l is always very great in proportion to r; we will call this quantity, which may be neglected in almost all cases, the "*missing quantity*," and will give it special consideration in another chapter. Neglecting this quantity, the formula becomes

$$(I) \qquad \xi = r \sin\delta \cos\omega + r \cos\delta \sin\omega.$$

Now we may state here, that it is a peculiarity of all slide-valve motions, the link motion as well as the simple motion, that the formula for the movement ξ of the valve (measured either to the right

* [Since $\cos^2\beta - \cos^2\alpha = \sin(\alpha + \beta)\sin(\alpha - \beta)$, we get by substituting δ for β and $\omega + \delta$ for α: $\cos^2\delta - \cos^2(\omega + \delta) = \sin(2\delta + \omega)\sin\omega$.—Ed.]

or to the left of the centre of the movement) can be always reduced to the general form of $\xi = A \cos \omega \pm B \sin \omega + F$, in which A and B represent values which depend upon several dimensions of the gear, but are independent of the angle ω through which the axle is turned, and which are therefore constant quantities which may be calculated easily for the simple valve motion, according to equations (7) and (8). The missing quantity F depends upon the amount of angular movement; but the numerical value of it is, in correct and well-constructed valve-gears, so very small, that it may be almost always neglected. The movement of the valve is then, in general,

$$(\mathrm{I}^{\mathrm{d}}) \qquad \xi = A \cos \omega \pm B \sin \omega.$$

In investigating the formula (I^{d}) more minutely, we shall at first take no notice of the special values of A and B, and only use farther on the values $A = r \sin \delta$, $B = r \cos \delta$, which we have already found for the simple valve-gear. As we shall at a future time meet with valve motions in which B is negative, we shall examine—as already indicated by the manner of writing equation (I^{d})—in the following discussion, the meaning of both signs.

The equation (I^{d}) above given is *the polar equation of two circles of equal diameter which touch each other, it being supposed that the pole lies at the point of contact.*

In order to prove this, let O be the touching point of two circles, Fig. 3, both described with the radius $OC = \rho$, and let OX be any straight line passing through the point of contact O; further, let $OB = a$ and $CB = b$ be the co-ordinates of the centre of one circle; then, if we express the radius vector OP of any point P of the circle by ξ, and the angle formed by this line OP with the original position OX by ω, we have

$$OM = \xi \cos \omega, \text{ and } MP = \xi \sin \omega.$$

If we now draw the line CN parallel to OX, we get

$$\overline{CN}^2 + \overline{NP}^2 = \overline{CP}^2,$$

or,

$$(OM - OB)^2 + (MP - MN)^2 = \overline{CP}^2,$$

which gives

$$(\xi \cos \omega - a)^2 + (\xi \sin \omega - b)^2 = \rho^2;$$

whence it follows that as $a^2 + b^2 = \rho^2$,

$$\xi = 2\,a \cos \omega + 2\,b \sin \omega.$$

FIG. 3.

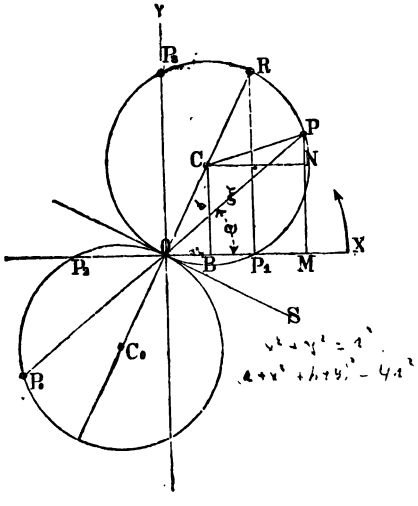
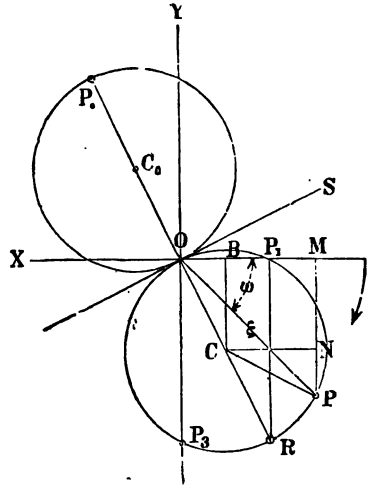


FIG. 4.



If the ordinate $BC = b$ of the centre of the circle extends from B downwards, then, the centre C being below O X (Fig. 4), b becomes negative, and

$$\xi = 2\,a \cos \omega - 2\,b \sin \omega.$$

Both equations are identical with equation (I^d), if

$$2\,a = A \text{ and } 2\,b = B;$$

whence follows

$$a = \frac{A}{2} \text{ and } b = \frac{B}{2}.^* \qquad (9)$$

* The equation (I^d) is, mathematically, the polar equation of a *single* circle, described in Figs. 3 and 4 from C. This will be easy to prove, if the equation of the curve referred to rectangular co-ordinates is taken instead of the polar equation. If we put, in Fig. 3, $OM = x = \xi \cos \omega$ and $MP = y = \xi \sin \omega$, it follows from equation (I^d):

$$x^2 + y^2 = A\,x + B\,y,$$

and this equation gives for each value of x only two values of y. This also confirms the polar equation (I^d) itself; for by substituting in the equation

$$\xi = A \cos \omega + B \sin \omega$$

If, therefore, the values of A and B are known for a certain case, the centre C of the circle may be at once determined, and this serves to determine the movement of the valve, if the distance O B upon O X measured from O is made $= a = \frac{A}{2}$ and the perpendicular at B, or $BC = b = \frac{B}{2}$. The radius of the circle is found without any further difficulty:

$$\rho = \sqrt{a^2 + b^2} = \frac{1}{2}\sqrt{A^2 + B^2}. \qquad (10)$$

It will be seen from the above, with what facility the valve movement ξ may be obtained for any angle $XOP = \omega$ of the crank: *this valve movement is simply the radius vector O P corresponding to that angle.* If, therefore, the centre of the corresponding circle passing through O can be fixed, which is always very easy, as will be seen from the following, the movement of the valve, or the distance between the valve centre and the centre of movement for each position of the crank can be measured directly from the figure.

After the above statement, we may examine immediately a few general relations between the movement ξ of the valve and the angular movement ω, considering that the values of A and B are still certain constant ones, which depend upon the kind of valve-gear.

If the crank is at the dead point $\omega = 0$, it follows from the equation

$$\xi = A \cos \omega + B \sin \omega,$$

that

$$\xi = A;$$

or as, according to the above, $OB = a = \frac{A}{2}$ (Fig. 3),

$$\xi = OP_1.$$

$\omega + 180°$ for ω, it follows

$$\xi = -(A \cos \omega + B \sin \omega);$$

the radius vector OP_0, in Fig. 3, must therefore not be drawn from O in the direction OP_0, but in the negative direction, i. e. again towards P, and therefore leading again to the circle described from the point C. The admission, however, that we have to deal with two circles touching each other, facilitates greatly the study of the laws according to which the movement of the slide-valve takes place in the different valve-gears.

The line O P_1 *therefore gives at once the movement of the valve at the beginning of the stroke of the piston.* If $\omega = 180°$, i. e. if the crank is at the other dead point, then

$$\xi = -A,$$

or, according to the figure, $\xi = O P_2$, where P_2 signifies the point at which the second circle cuts the line O X. The movement of the valve is = *nil*, or the valve is in its central position if

$$0 = A \cos \omega \pm B \sin \omega,$$

or if

$$\tan \omega = \mp \frac{A}{B} = \mp \frac{a}{b}.$$

The upper sign is taken if the centre of the circle lies above O X (Fig. 3).

But from the condition

$$\tan \omega = -\frac{a}{b},$$

it appears that the corresponding angle may be very easily found if a line O S is drawn through O and at right angles to O C; the angle S O X shows therefore the angle at which the crank stands *before* the dead point, when the slide-valve is in the central position. If the centre C lies below O X, as really happens in some valve-motions, which will be examined later, it is

$$\tan \omega = +\frac{a}{b};$$

in this case also the required angle S O X is found by dropping the perpendicular O S from O upon O C (Fig. 4). The maximum movement of the valve ξ is found from the equation:

$$\xi = A \cos \omega \pm B \sin \omega;$$

by differentiation

$$\frac{d\xi}{d\omega} = -A \sin \omega \pm B \cos \omega = 0,$$

or

$$\tan \omega = \pm \frac{B}{A} = \pm \frac{b}{a},$$

i. e. the corresponding angle ω is that which the line O C (Figs. 3 and 4) forms with O X, or C O X; the valve movement itself is in this case

$$O R = 2 \rho = \sqrt{A^2 + B^2}.$$

If finally the crank is at right angles to the line of motion of the piston, i. e. if it has moved 90° from the dead point,

$$\xi = A \cos 90° \pm B \sin 90° = \pm B.$$

If therefore, in Fig. 3 or 4, $O P_3$ is perpendicular to $O X$, $O P_3 = B$ indicates the position of the valve when the crank is at right angles to the direction of the motion of the piston.

By all this it appears that it is only necessary, in order to ascertain the distance of the valve from the centre of its stroke for different positions of the crank, to describe *one* circle from the point C; if $O P$ is produced above O as far as P_0, then $O P_0 = O P$; as now $O P_0$ expresses the distance of the valve from the centre of its stroke for $(180° + \omega)$, it is only necessary on account of the equality of $O P$ and $O P_0$ to fix $O P$, or to describe one circle. In the same manner the case represented in Fig. 4, which occurs under the supposition that the second quantity in the equation, 2nd member of the equation

$$\xi = A \cos \omega \pm B \sin \omega,$$

is taken as negative, may be easily referred to the case shown in Fig. 3, the two cases differing only in that the angular movement of the crank takes place in opposite directions. The angles ω are in Fig. 3 to be laid off from $O X$ upwards, and in Fig. 4 from $O X$ downwards. If A and B are in both instances equal, the same will be the case with $O P$ for equal angles ω. It is therefore not necessary for us to take any further notice of the sign of the second quantity of the above equation in the following general investigation.

We shall now apply the results above obtained to the simple valve-motion, in which, according to equations (7) and (8),

$$A = r \sin \delta,$$
$$B = r \cos \delta.$$

The co-ordinates of the centre of that circle whose chords give directly the movement of the valve (Fig. 3, p. 16) are, according to equation (9),

$$O B = a = \frac{A}{2} = \frac{1}{2} r \sin \delta,$$

and

$$B C = b = \frac{B}{2} = \frac{1}{2} r \cos \delta,$$

therefore the radius of this circle, which we will now call the "*valve circle*," is

$$CO = \rho = \frac{1}{2}\sqrt{A^2 + B^2} = \frac{1}{2}r,$$

and its diameter

$$OR = 2\rho = r;$$

i. e. in the simple valve-motion *the diameter of the valve circle is equal to the eccentricity r.*

The angle C O Y, formed by the diameter of the valve circle with the vertical O Y, is next found (Fig. 3):

$$\tan COY = \frac{OB}{BC} = \frac{a}{b} = \frac{A}{B} = \tan\delta,$$

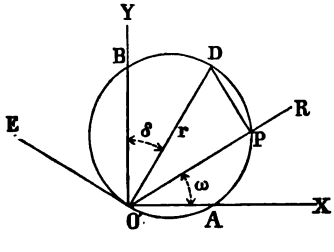
whence $$\angle COY = \delta,$$

or in words: *the required angle is equal to the angle of advance.*

From what has been already stated, it will be seen with what facility the law of the slide-valve motion, if the valve is driven by an eccentric, may be laid down graphically. Had the problem only been to find the law of the movements of the simple slide-valve gear with one eccentric, the results of the calculation last given might have been obtained in a much shorter way, as follows:—

Let O Y in Fig. 5 be a perpendicular let fall from the point O upon O X, the direction of the valve-face; then draw from O a line O D = r = the eccentricity, this forming with the perpendicular O Y the angle Y O D = δ the angle of advance; and next describe, with O D = r as diameter, a circle, which will be the valve circle. Let the crank be in the position O X, or on its dead point, and from there let it be turned through the angle X O R = ω, then the length O P gives at once the distance of the valve from its central position when the crank angle = ω.

Fig. 5.



Neglecting the "missing quantity," equation (I^a) gives the valve movement

$$\xi = r \sin(\omega + \delta).$$

Drawing in Fig. 5 the line D P, the angle D P O is a right angle, while the angle D O P in triangle D O P is found to be $90° - (\omega + \delta)$, and since O D $= r$, we at once see that

$$O P = r \cos[90° - (\omega + \delta)],$$

or

$$O P = r \sin(\omega + \delta) = \xi,$$

which was to be proved.

If the equation for the movement of the valve is given in the form

$$\xi = A \cos \omega + B \sin \omega,$$

as we above supposed, and as is the case in a number of valve-motions which we shall examine later on, and in which the constant values of A and B are not only dependent upon r and δ but also upon the dimensions of other parts of the valve-gear, then make in Fig. 5, O A $=$ A and O B $=$ B, and draw a circle through the points O, A, and B; such a valve-motion, whatever the nature of its mechanism, then works exactly like a simple motion, in which the eccentricity of the eccentric O D $= r$ and the angle of advance Y O D $= \delta$. If the movement of the valve is represented by the last-given equation, then the valve-motion may always be reduced in the manner indicated to a simple valve-motion; a fact of which we shall make use hereafter.

Besides this, the simple diagram also gives the means of representing the velocity of the slide-valve. The velocity of the slide-valve varies, and the manner in which this variation takes place may be seen from diagram Fig. 5, by noticing how the radius vector O P alters with the angle X O R $= \omega$. It is evident that the valve moves very slowly, in fact almost stands still, when it is near the end of its stroke, i. e. when the crank stands near the direction O D; but on the other hand, when the crank is in the position O E (perpendicular to O D), the valve has its greatest velocity, and this occurs when it is near the middle of its travel or stroke.

The law which governs the variations of velocity, is also very easily demonstrated in a graphical and analytical manner. Let the

crank turn with the constant velocity ϵ; then if, at the moment in which the crank passes through the dead point, we begin to count the time t, we have

$$\omega = \epsilon t,$$

whence follows by substitution in the general formula for the travel of the valve

$$\xi = A \cos \epsilon t + B \sin \epsilon t,$$

and from this the velocity v of the valve at the time t,

$$v = \frac{d\xi}{dt} = -A \epsilon \sin \epsilon t + B \epsilon \cos \epsilon t,$$

or

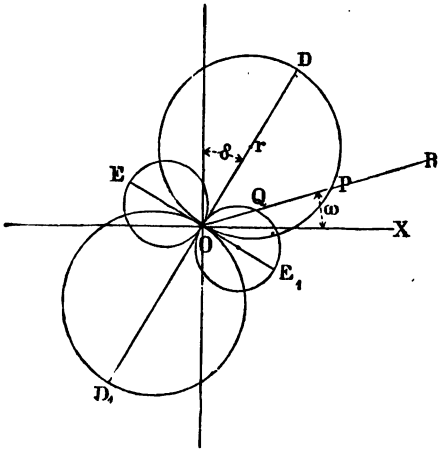
$$v = -A \epsilon \sin \omega + B \epsilon \cos \omega,$$

from which the velocity of the valve may be calculated for any angular position of the crank. If the valve-motion is considered to be reduced to a simple one, then put $A = r \sin \delta$ and $B = r \cos \delta$, and the velocity of the valve becomes

$$v = -r \epsilon \sin \delta \sin \omega + r \epsilon \cos \delta \cos \omega,$$

and this is the polar equation of two circles O E and O E_1 (Fig. 6), whose diameter $r \epsilon$ is perpendicular to the centre line of the eccentric O D $= r$. At the position O R of the crank, i.e. when the angular motion $= \omega$, the movement of the valve from its central position is O P, while the radius vector O Q represents the *velocity* v of the valve at that moment. The law for the variations of velocities can now be seen clearly; at the positions of the crank O D and O D_1 the velocity is *nil*, i.e. the valve stands still momentarily (at the greatest distance from its central position), and at the positions O E and O E_1 of the crank (at half-stroke) the valve has its greatest velocity $r \epsilon$.

FIG. 6.



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PRACTICAL APPLICATION OF THE DIAGRAM.

We shall at first take an existing simple valve-motion and show the manner in which the distribution of steam is to be ascertained from the dimensions of the parts of the valve-gear.

The amount of eccentricity O D (Fig. 2) $= r$, as well as the angle D O Y $= \delta$, may be taken as given. With these values as a base, we shall first describe the movement of the valve and show how far it is from its central position for any given angular movement ω of the crank from its dead point. This distance or movement of the valve is designated by ξ.

Draw the system of axes O X and O Y (Fig. 2, Pl. I.), O X being supposed to coincide with the direction of the valve-rod or of the valve-face. Draw from O a line O Z, so that Y O Z $= \delta =$ angle of advance, and make part O P_0 of O Z, as well as part O Q_0 of the prolongation of O Z, equal to the given eccentricity r; on these parts as *diameters* describe circles; these are the so-called *valve circles* which explain the movement of the valve in the following manner:—

If we suppose a crank to occupy the position O X or O R, when the actual crank just passes through the first of its dead points, then if both cranks are turned through the angle ω, so that the imaginary crank comes into the position O R_1, that part O P of the line O R_1 which lies inside the valve circle, at once represents the movement of the valve $= \xi$; i.e. the distance through which the valve has moved from its central position while the actual crank has travelled through the angle ω from its dead point. The figure gives for the angular movement X O Q $= 180° + \omega$ the same value for ξ, but the value is in this case negative; for the centre line of the crank here cuts the second circle described from C in such a manner that O Q = O P. The *upper* circle represents therefore the movement of the valve to the *right* of its central position (see Fig. 1, Pl. I.), the *lower* circle that to the *left*.* The results for both directions are the same,

* [To determine whether the valve is to the *right* or *left* of its central position while the imaginary crank is intersecting a particular valve circle, we lay off the eccentricity in advance of the imaginary crank and notice whether the eccentricity is respectively to

and we have therefore only to examine the upper circle. Before proceeding any further with our investigations, we may state that the figures shows everything *full size*, and that the eccentricity is supposed to be taken as $O P_0 = r = 0 \cdot 060^m$ (almost exactly $2\frac{11}{32}''$), and the angle of advance $Y O P = \delta = 33°$. In order to ascertain, therefore, how far the valve has moved to the right of its central position at the corresponding position of the crank, it is only necessary to measure the distance O P. If the crank stands at the dead point, it occupies the position O R, and then the distance of the valve from its central position is equal to $O P_1$; in the present case this distance $O P_1 = 0 \cdot 030^m$ (almost $1\frac{3}{16}''$).

If the crank has turned so far that the chord O P becomes equal to the diameter, then the valve is farthest from the centre of its stroke; it will be seen from the figure that such is the case when the crank is in the position $O P_0$, or as moved from the dead point through the angle $P_0 O X = 90° - \delta$. A perpendicular $O R_0$ dropped upon the line $P_0 Q_0$ at the point O does not cut the circle, and the chord O P becomes therefore = *nil*, the valve being just in the centre of its stroke when the crank occupies the position $O R_0$, i. e. when it stands as much as the angle $R O R_0 = \delta$ before the dead point. Suppose the crank is turned from O R in the direction of the arrow, it will easily be seen by the figure that the chords which represent the movements of the valve increase quickly from O, while they vary but little when the crank is near the position $O P_0$. The slide-valve travels therefore very quickly towards the middle of its

---

the *left* or *right* of the vertical O Y. However, when the diagram has once been drawn for a particular case, there will be less liability to confusion and error if we dispense with the imaginary crank by supposing the engine viewed from the side which shows the actual crank rotating in the same direction as the radius vector (see Figs. 3 and 4); for example, when the engine shown in Fig. 1, Pl. I. is viewed from the side opposite to that given in the figure, the actual crank and piston directions will be exactly like those indicated by the arrows in the diagrams of Fig. 2, Pl. I.; when the engine is thus viewed the *upper* valve circle represents the movement of the valve to the *left* of its central position.

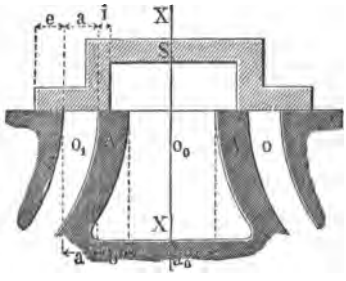
Some writers have the actual crank directions intersect the valve circle by drawing the latter on that side of the vertical O Y which is opposite to that given by Prof. Zeuner; this, however, requires that the polar co-ordinates be estimated differently from what is customary, and is therefore liable to lead to confusion in the analytical investigations connected with valve-gears.—Ed.]

stroke; but its movement takes place very slowly near the ends of the latter. This fact, known long ago, is of great importance, and it exercises great influence on the whole distribution of the steam. The figure plainly shows also the influence of the angle of advance, and of the amount of the eccentricity upon the movements of the valve corresponding to certain positions of the crank. If, for example, the eccentric were fixed without any angular advance, δ would $= 0$, and the diameter $O P_0$ would fall upon the vertical line O Y. If therefore the crank were on its dead points, the slide-valve would be just in its central position, and it would be most remote from the latter when the crank had turned through the angle 90° and 270°. The following will show that the distribution of steam thus effected would be unsuitable, and therefore such a case seldom occurs.

If we take $\delta = 90°$, $O P_0$ will fall on the line O X; i.e. the valve will be farthest from its central position when the crank stands at its dead points, and the valve will occupy its central position when the crank has moved through 90° and 270°. Observations respecting the alterations in the movement of the valve may be made with equal facility, if the eccentricity r alone, or r and δ simultaneously, be altered.

Now in practical investigations the main problem is not to ascertain the valve movement for different crank angles, but to ascertain the *distribution of the steam* when certain dimensions of the valve-gear are given, or, *vice versâ*, to determine the valve-gear dimensions when the distribution of the steam is assumed. The dimensions referred to include not only the eccentricity r and angle of advance δ, but also dimensions of the valve itself; these we will now examine.

Fig. 7.

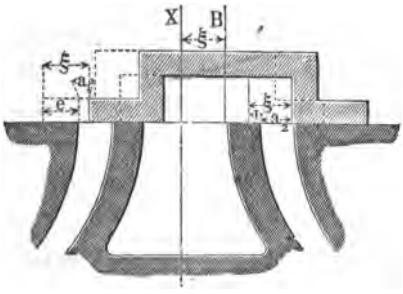


In Fig. 7 the valve is shown in its middle position, i.e. when the valve centre is just passing the centre of motion X. The two admission-ports o and o_1 are separated from the exhaust-port o_0 by the fixed bars or bridges A A.

It will be seen that the face of the slide-valve not only completely shuts the two steam-ports, but that it extends at each end for a certain amount beyond the openings. This amount e is called *outside lap*, or more frequently simply *lap*. On the other hand, the face extends also partly over the bridges A for a distance i; this amount is called the *inside lap*. The outside and inside laps, and the width a of the steam passages o and o_1, are the additional (valve) dimensions which we must introduce into the following investigations.

If the valve has moved as much as ξ (Fig. 8), and if we have thereby obtained the opening of the port for the admission $= a_1$, we at once get, as will be seen from the figure, the equation

FIG. 8.



$$e + a_1 = \xi,$$

or the opening of the port is

$$a_1 = \xi - e,$$

if the outside lap e is known. But the valve has at the same time opened the other port as much as a_2 for the discharge of the steam, and we get in this case, as is also shown by the figure, the relation

$$i + a_2 = \xi,$$

or

$$a_2 = \xi - i.$$

These two equations for a_1 and a_2 at once give the size of the openings for the admission and for the release, as soon as there is fixed, according to the method above explained, the movement of the valve for any angular movement ω of the crank. It is only necessary to subtract in the one case the outside lap e, and in the other case the inside lap i, from the movement of the valve to get the required opening. If it is desired to find these openings by construction, then they may be obtained in a simple manner, as follows:—

We have found that for any angular movement $XOP = \omega$ (Fig. 2, Pl. I.), the movement of the valve is equal to the radius

vector O P. Now the size of the opening for the admission of the steam for this position is obtained by subtracting the outside lap e from ξ; and therefore by describing from the point O a circle $V_1 V V_2$ with the radius $O V = e$, the distance P V gives *at once the opening of the steam-port* corresponding to the angular movement $P O X = \omega$.

Describing in the same manner from O, with the radius $O W = i$, the circle $W_1 W W_2$, it follows that the part P W of the radius vector, or $O P - O W = \xi - i$; in other words, it is equal to the width of the opening of the other port for the exhaust.

As the figure is drawn full-size, and as the eccentricity r is supposed to be $= 0 \cdot 060^m$ ($O P_0$), almost $2\frac{11}{32}''$, the outside lap $O V = e = 0 \cdot 024^m$ ($\cdot 94''$), the inside lap $O W = i = 0 \cdot 007^m$ ($\cdot 27''$), and the angle of advance $Y O P_0 = \delta = 30°$, the dimensions O P, P V, P W, may be taken directly from the figure.

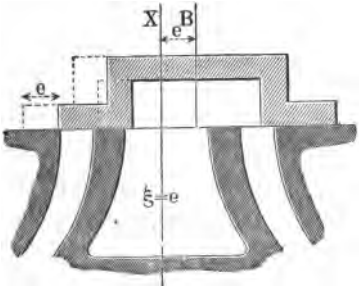
If the crank stands at a dead point, for instance at O R, so that the piston is at the *beginning of its stroke*, the travel of the valve $= O P_1$, the opening for the exhaust is $P_1 W_1$, and the opening for the admission of the steam is $P_1 V_1$. As is known, the opening of the port for the admission of the steam at the beginning of the stroke of the piston, or $V_1 P_1$ is called *the outside lead*, or the lead on the steam side; but the opening of the other port for the exhaust at the beginning of the stroke of the piston, or $W_1 P_1$, is called *the inside lead*, or the lead on the exhaust side.

For our special case, as may be immediately ascertained by measurement, the *outside* lead $V_1 P_1 = 0 \cdot 006^m$ ($\cdot 236''$), the *inside* lead $W_1 P_1 = 0 \cdot 023^m$ ($\cdot 905''$). The largest opening of the steam-ports occurs when the valve has travelled the greatest distance from its central position, i. e. when the crank stands in the position $O P_0$; therefore the *largest* opening of the port for the admission of the steam is $P_0 V_5$ ($0 \cdot 36^m = 1 \cdot 417''$), and the largest opening for the exhaust is $P_0 W_5$ ($0 \cdot 053^m = 2 \cdot 08''$), supposing the ports (or passages) themselves to have so great a width.

But Fig. 2, Pl. I., also answers all other questions. The question as to the positions of the crank when the admission of the steam begins and ends, can be answered in the following manner:—

At these moments the valve occupies the position shown in Fig. 9, where the valve has moved from its central position for a distance just equal to the outside lap e, i. e. $\xi = e$. If we now return to Fig. 2, Pl. I., we find that the principal circle round C cuts the circle described with the outside lap as a radius, in V_3 and V_4; and connecting both points with O and producing the connecting lines backwards to the circle which is described by the crank-pin, these lines will give the position of the crank at the two moments at which one outer edge of the valve is just at the outside of the port, or, in other words, at which the admission commences and terminates.

FIG. 9.



Thus $O R_3$ is the position of the crank at the beginning of the admission of the steam; and it has therefore to pass through the angle $R_3 O R$ *before* reaching the dead point.

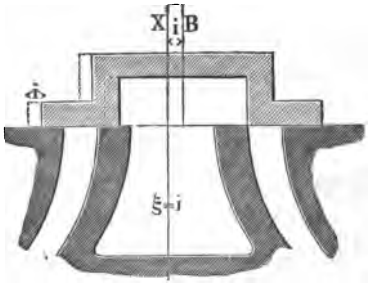
Moreover, $O R_4$ is the position of the crank at the end of the admission of the steam, and the angle $R_3 O R_4$ shows at the same time the arc through which the crank passes during the admission.

It is more convenient to know the positions of the piston corresponding to these positions of the crank. To ascertain this, drop from R_3 and R_4 perpendiculars upon O X. In order to avoid covering the figure with too many lines, a line H K has been drawn parallel to O X, its length being made equal to the diameter of the crank-pin circle, and consequently to the stroke of the piston. Letting fall now from R_3 and R_4 the perpendiculars $R_3 H_3$ and $R_4 H_4$ upon H K, H_3 gives us the position of the piston at the beginning of the admission of the steam, and H_3 K the distance of the piston at that moment from the end of its stroke; H_4 also gives the position of the piston at the end of the admission of the steam; the ratio between the distances $K H_4$ and K H *represents therefore the degree of expansion or cut off.*

The questions with regard to the exhaust can be answered in an equally ready manner. Fig. 10 represents the valve in the position which it occupies when the exhaust is either just beginning or just ending, the valve having in this case travelled the distance $\xi = i$

from its central position. From Fig. 2, Pl. I., it may be seen that this equality of ξ and i takes place in the position corresponding to the two intersections W_3 and W_4 of the circles $P_1 P P_2$ and $W_1 W W_2$. Connecting therefore W_3 and W_4 with O, and producing this line backwards until it meets the circle described by the crank-pin, we get at $O R_5$ the position of the crank, and at H_5 the position of the piston at the beginning of the exhaust. The exhaust therefore begins when the crank stands at the angle $R O R_5$ *before* the dead point, or when the piston has still the distance $H_5 K$ to travel before reaching the end of its stroke.

FIG. 10.



$O R_6$ gives the position of the crank at the end of the exhaust and H_6 the position of the piston at that moment. The ratio of $K H_6$ to $K H$ represents the percentage of the stroke occupied by the exhaust.

It is readily seen that while the crank passes through the angle $R_4 O R_6$, or while the piston travels the distance $H_4 H_6$, the steam expands in the cylinder end nearest the shaft and exhausts from the cylinder end which is farthest from shaft. When the crank has reached the position $O R_6$, the compression of steam will commence in the cylinder end farthest from shaft.

While the crank-pin is moving along the arc $R_6 R_7$, expansion takes place in the cylinder end nearest the shaft, and compression in the end farthest from shaft.

If $O R_3$ is produced as far as R_8, $O R_8$ will represent the position occupied by the crank when the admission of steam begins at the cylinder end farthest from shaft; therefore, while the crank-pin passes through the arc $R_6 R_8$, compression of the steam takes place in the cylinder end farthest from shaft.

All that has been said applies to the stroke of the piston *away from* the crank-shaft, and for the investigation of the opposite stroke *

* [As the exhaust from one end of the cylinder begins shortly before the commencement of a new stroke and ends after this stroke is well advanced, we refer this exhaust to the stroke during which it mainly takes place. This is also true of the admission. Bearing this in mind, the above paragraph will be found to contain no contradictions.—ED.]

it is only necessary to produce the chief centre lines of the crank $O R_5$, $O R_3$, $O R_4$, $O R_6$ past O until they again meet the crank-pin circle. Thus R_7 lies on the prolongation of $O R_5$, and $O R_7$ represents the position of the crank at the beginning of the exhaust from the cylinder end *nearest the crank-shaft* when the piston is moving *away from* this shaft.

In order that the figure may be understood more readily, all the different occurrences are inscribed on it; the explanations outside, around the crank-pin circle, relate to the distribution of steam in the cylinder end *nearest* the crank-shaft, whilst the explanations written inside the circle relate to the distribution in the cylinder end *farthest* from crank-shaft.

In the manner already stated, i. e. by dropping from the different positions of the crank-pin perpendiculars upon the lines H K and $H_0 K_0$, the positions of the piston shown upon the first of these lines are those corresponding to the motion of the piston away from the crank-shaft, whilst upon the latter line are marked the positions corresponding to the motion towards the crank-shaft.

In fixing the positions of the piston in the above manner, no notice has been taken of the influence of the length of the connecting-rod; if, however, it is desired to allow for this influence, it is only necessary to consider that $R_1 R R_4$ is the crank-pin circle, and $O X_1$ the direction of the piston-rod. In proceeding therefore in an exact manner, we have to take the length of the connecting-rod as a radius, and with the different points R, R_1, R_4, R_6 . . . . as centres, describe arcs cutting the line $O X_1$ produced. The points of intersection then give the positions of the piston exactly for the chief positions of the crank. For want of space, we have chosen on the Plate the more simple but less exact method of fixing the positions of the piston.*

It is scarcely necessary to point out how easily the influence of

* [Without taking up any more space, we can, by multiplying the tabular quantities given on pp. 8 and 9 by the diameter of the crank-pin circle assumed in the diagram, get exactly the piston positions corresponding to the principal crank positions and to the given ratio of connecting-rod to crank.—Ed.]

the inside and outside lap, of the angle of advance, &c., &c., may be perceived from Fig. 2, Pl. I. For example, make the outside lap smaller than O V, then the circle, described with a radius equal to the new amount of outside lap (not shown in the figure), will cut the circle described from the centre C in points of the circumference of the latter which are nearer to O than V_3 and V_4. Again connecting both points of intersection with O, the two connecting lines will form a larger angle than O V_3 and O V_4, i. e. admission of steam will take place during a longer time and the expansion will be less.

Let it be supposed, as another example, that there is no inside lap at all, the circle W W_2 W_4 will then disappear altogether, and a continual exhaust take place.

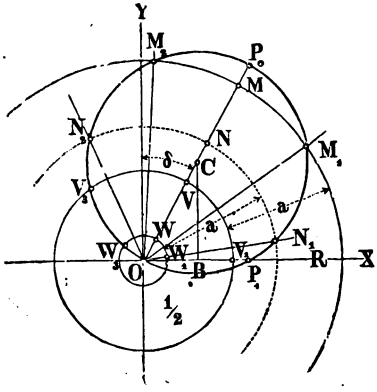
In order that, at the beginning of the stroke of the piston, steam may already be on the driving side of the latter, the port for the admission of the steam must at that moment be open, and we have seen that V_1 P_1 at once gives the size of this opening. It will be seen from the figure that in order that this condition may be fulfilled, O V_1 must be less than O P_1, i. e. the outside lap must be smaller than O P_1.

If no outside lap were given, a continual admission of steam would take place, and the admission would always change at the moment when the valve passed the middle of its stroke, or when the crank was in the position O R_0.

The alterations which occur in the distribution of the steam, if some of the dimensions r, δ, e or i are altered separately or together, will be so easily seen from the simple diagram, that it is unnecessary that we should refer to it further here. We may, however, make one remark as regards the *width of the steam-ports;* viz. that it is very easy to fix, by means of the diagram, the moments at which the ports are fully open for the admission or for the exhaust of the steam, or at which the ports just begin to close. Fig. 11 shows half-size the principal parts of the diagram Fig. 2, Pl. I. Lay out from V on O P_0 the distance V M = a = the width of a steam-port (which is in this case $a = 0 \cdot 030^m$, or $1 \cdot 18''$), and also from W the same distance W N, so that W N is also = a. Next describe from O circles with

O M and O N as radii, and it will be easy to understand the correctness of the following: Take the points where these circles cut the valve circle, and then O M_1 will be the position of the crank when the port for the admission of steam is just fully open, and O M_2 the crank position when the port just begins to close. While the crank passes through the arcs M_1 O M_2, the admission port is wide open. M P_0 shows, half-size in Fig. 11 and full-size in Fig. 2, Pl. I., how far the outer edge of the valve has gone beyond the inner edge of the port at the time of largest opening.

Fig. 11.



The same process is applicable to the exhaust. When the crank is in the position O N_1 the port is fully opened for the exhaust, and it remains so until the crank arrives at the position O N_2.

It has been objected to the diagram that, for the positions in which the valve is near the middle of its stroke, the crank directions intersect the valve circle at very small angles, and therefore do not determine the points of intersection with sufficient exactness. The objection is unfounded. For example, if the crank positions O R_4 and O R_6 (Fig. 2, Pl. I.) for the beginning of expansion and compression were given, and if the intersections V_4 and W_4 were to be found for the purpose of determining the outside lap O V_4 and inside lap O W_4, then the figure shows that the intersections do take place at very small angles; but in such a case it is only necessary to let fall, from the extremity P_0 of the valve-circle diameter O P_0, the perpendiculars P_0 V_4 and P_0 W_4 on the directions O R_4 and O R_6, and then the resulting intersections V_4 and W_4 will be determined with sufficient exactness.

The preceding investigation shows how all the details in the distribution of the steam, effected by a simple valve-motion, may be quickly and correctly ascertained by the diagram; and it only

remains for us to show how to determine the dimensions for a valve-gear so as to satisfy certain conditions with regard to the distribution of the steam.

Solutions of a few Problems.

Problem 1.—"In a simple slide-valve gear let the eccentricity $r = 0 \cdot 060^m$ ($2 \cdot 3''$) and the angle of advance $\delta = 30°$. Let the admission of steam take place whilst the piston travels $0 \cdot 8$ of its stroke, and let the exhaust begin when the piston has still $0 \cdot 04$ of its stroke to travel. It is required to find the inside and outside lap, the inside and outside lead, the greatest opening of the steam-ports, &c., &c."

Solution.—Draw the two axes O X and O Y (Fig. 3, Pl. I.) perpendicular to each other; lay off from O Y the angle $Y\ O\ P_0 = \delta = 30°$, and make $O\ P_0 = r = 0 \cdot 060^m$ ($2 \cdot 36''$); bisect $O\ P_0$ in C, and describe from C, with $O\ C = C\ P_0$ as a radius, the *valve circle.* Next describe from O, to any scale, the circle $R\ R_4\ R_0$, which represents the path of the crank-pin. In the diagram we have taken $R\ R_0 = 0 \cdot 100^m$. Now if we suppose that the crank turns in the direction of the arrow, and that the piston travels away from the crank-shaft (Fig. 1, Pl. I.), the exhaust will have to begin in the cylinder end farthest from crank, whilst the piston is still distant from the end of its travel as much as $0 \cdot 04$ of its stroke, thus in the figure this distance will be $0 \cdot 04$ multiplied by $0 \cdot 100 = 0 \cdot 004^m$ ($\cdot 15''$). Therefore, if we make $R\ H_3 = 4^{mm}$ ($\cdot 15''$), erect the perpendicular $H_3\ R_3$ meeting the crank-pin circle in R_3 and draw $O\ R_3$, this line will represent the position of the crank *before* the dead point at which the exhaust begins. The line $O\ R_3$ cuts the valve circle at W_3, and $O\ W_3$ is the required inside lap.

It is required that the admission of the steam shall cease when the piston has passed through $0 \cdot 8$ of its stroke. Making therefore $R\ H_4 = 0 \cdot 8 \times R\ R_0 = 0 \cdot 8 \times 0 \cdot 1 = 0 \cdot 08^m$ ($3 \cdot 15''$), and erecting the perpendicular $H_4\ R_4$, we get by drawing the line $O\ R_4$ the position of the crank at the end of the admission of the steam, or at the beginning of the expansion. This line $O\ R_4$ cuts the valve circle at V_4, so that $O\ V_4$ gives at once the required outside lap e. We must next

describe from O circles with the inside lap $O W_3 = i$ and the outside lap $O V_4 = e$ as radii, and the whole problem is solved, for we have at once all the required quantities; viz. if we put together the measurements as taken from Fig. 3, Pl. I.:

Outside lap	$O V_4$	$= 0 \cdot 024^m$ $(0 \cdot 94'')$
Inside lap	$O W_3$	$= 0 \cdot 007^m$ $(0 \cdot 27'')$
Outside lead	$P_1 V_1$	$= 0 \cdot 006^m$ $(0 \cdot 23'')$
Inside lead	$P_1 W_1$	$= 0 \cdot 023^m$ $(0 \cdot 903'')$
Greatest opening of the port for admission	$P_0 V$	$= 0 \cdot 036^m$ $(1 \cdot 42'')$

At the same time, answers, as with Fig. 2, Plate I., may be obtained to all other questions regarding the distribution of steam and the positions of the piston, but we cannot repeat them here.

To find a general and analytical solution of this problem, it would be necessary to proceed as follows: Let s_1 be the travel of the piston during the admission, i. e. up to the beginning of the expansion, R = the radius of the crank, and ω_1 = the angle through which the crank has turned, measured from the dead point up to the beginning of the expansion, then there exists (supposing that there is a long connecting-rod) the relation

$$s_1 = R\,(1 - \cos \omega_1),$$

but the full travel of the piston is

$$s = 2\,R.$$

From this follows, by division, the degree of expansion

$$\frac{s_1}{s} = \frac{1 - \cos \omega_1}{2} = \sin^2 \frac{1}{2}\,\omega_1,$$

and the angular movement ω_1 up to the beginning of the expansion is therefore calculated from the degree of expansion according to the formula

$$\sin \frac{1}{2}\,\omega_1 = \sqrt{\frac{s_1}{s}}\,.$$

Next let the travel of the piston up to the beginning of the exhaust be represented by s_3, and the corresponding angular movement of the crank by ω_3, we then get in the same manner

$$\sin \frac{1}{2} \omega_3 = \sqrt{\frac{s_3}{s}}.$$

But the movement of the valve for the angle ω is, in general,

$$\xi = r \sin (\omega + \delta).$$

At the beginning of the expansion it is $\xi = e$, as was found by the diagram, and at the beginning of the exhaust, i. e. for $\omega = \omega_3$, it is $\xi = -i$, and we obtain at once (as r and δ are considered as known), for the calculation of the outside and inside lap, the two formulas

$$e = r \sin (\omega_1 + \delta) \quad \text{and} \quad i = -r \sin (\omega_3 + \delta).$$

At the beginning of the stroke $\omega = 0$, and therefore the travel of the valve $\xi = r \sin \delta$, whence may be calculated the outside lead $= r \sin \delta - e$ and the inside lead $= r \sin \delta - i$.

Problem 2.—"The radius r of the eccentricity and the angle of advance δ are to be determined for a slide-valve gear with one eccentric, when the dimensions of the valve (i. e. the inside and outside lap, or i and e), the degree of expansion $\frac{s_1}{s}$, and the opening of the entrance-port at the beginning of the stroke of the piston (i. e. outside lead v), are given. For instance, let e be taken $= 0 \cdot 024^m$ ($\cdot 94''$); $i = 0 \cdot 007^m$ ($\cdot 272''$); $v = 0 \cdot 006^m$ ($\cdot 23''$), and $\frac{s_1}{s} = 0 \cdot 80$."

Solution.—The solution is reduced to the ascertaining of the position of the centre C of the valve circle (Fig. 3, Pl. I.). If this position is known, then C O $= \rho$ is the half of the required eccentricity, and the angle Y O C $= \delta$ is the angle of advance.

Let the point O be taken as centre of the axle, draw O X and perpendicular to it O Y; describe now from O, with the outside lap $e = 0 \cdot 024^m$ ($\cdot 94''$) and the inside lap $i = 0 \cdot 007^m$ ($\cdot 272''$) as radii, the circles V_1 V V_4 and W_1 W W_4; then describe also from O, to any other convenient scale, or preferably with the radius $0 \cdot 050^m$ ($1 \cdot 97''$) a circle, which represents the crank-pin circle (R R_4 R_0).

According to the problem, the distribution of steam shall be such that the degree of expansion $\frac{s_1}{s} = 0 \cdot 8$; in fixing therefore the point H_4 in the diameter $R\,R_0$ so that $R\,H_4 : R\,R_0 = s_1 : s$, H_4 will give the position of the piston at the end of the admission of the steam; as in the present case $s = 0 \cdot 100^m$ $(3 \cdot 93'')$ it is simply necessary to make $s_1 = R\,H_4 = 0 \cdot 080^m$ $(3 \cdot 15'')$; and drawing from H_4 the perpendicular $H_4\,R_4$, a line $O\,R_4$ will represent the corresponding position of the crank.

The line $O\,R_4$ cuts at V_4 the circle described with the outside lap, and that is, as is known from preceding remarks, a point through which the required valve circle passes.

According to the other conditions of the problem, the opening of the steam-port at the beginning of the stroke is to be $v = 0 \cdot 006^m$ $(\cdot 23'')$; since $O\,V_1 = e$ is known, we can lay off $V_1\,P_1 = v$ and then get at P_1 a second point through which the valve circle passes; this circle also passes through O; if therefore we describe a circle through the points V_4, O and P_1, it will be the required valve circle, and therefore $C\,O\,Y'$ will be the required angle of advance δ, and $O\,P_0$ the required eccentricity r. The measurements give $\delta = 30°$ and $r = 0 \cdot 060^m$ $(2 \cdot 36'')$.

By fixing the points where the valve circle cuts the other circles, we are able, as explained in the previous investigation, to answer any questions which may arise,

To solve this problem by calculation, we ascertain from the known degree of expansion, as in the first problem, the angular movement of the crank up to the beginning of the expansion, according to the following formula

$$\sin \frac{1}{2}\,\omega_1 = \sqrt{\frac{s_1}{s}};$$

next, the travel of the valve at the beginning of expansion

$$e = r \sin (\omega_1 + \delta),$$

and then, if the outside lead v is given, the travel of the valve at the beginning of the stroke of the piston

$$e + v = r \sin \delta.$$

By means of the last two equations the unknown quantities r and δ may easily be found.

The equation for determining the angle of advance δ is easily found to be

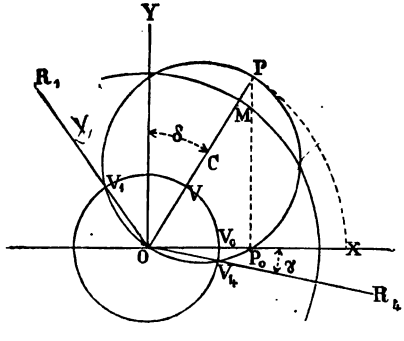
$$\cot \delta = \frac{e}{(e + v) \sin \omega_1} - \cot \omega_1,$$

and the eccentricity is given by the formula

$$r = \frac{e + v}{\sin \delta}.$$

Problem 3.—"Let there be given for the designing of a simple valve-gear the degree of expansion $\epsilon = \frac{s_1}{s}$, and *the angle of lead* γ, i. e. the angle R_4 O X (Fig. 12), at which the crank stands *before* the dead point at the *beginning of the admission of the steam.* Let there be also given the width of the ports V M $= a$ and the distance M P $= k$, at which the outer edge of the valve, at its greatest distance from the central position, stands beyond the inner edge of the admission-port.

FIG. 12.



"There is to be determined the eccentricity O P $= r$, the advance Y O P $= \delta$, the outside lap O V_1 = O V_4 $= e$, and the outside lead $V_0 P_0 = v$."

Solution.—In this and the following problem it is best to combine construction and calculation.

Fix at first the position of the crank O R_1, or the angle X O $R_1 = \omega_1$, for the beginning of the expansion, by means of the already explained method of construction, or according to the previously obtained formula

$$\sin \frac{\omega_1}{2} = \sqrt{\frac{s_1}{s}}.$$

Next lay off the angle of lead $X\,O\,R_4 = \gamma$, and bisect the angle $R_1\,O\,R_4$; the bisecting line O P gives at once the direction of the eccentricity, and the angle Y O P is therefore equal to the angle of advance δ.

The latter may be also found very simply from the formula

$$\delta = 90^\circ - \frac{\omega_1 - \gamma}{2}.$$

For the beginning of the expansion we must, in the general formula $\xi = r \sin(\omega + \delta)$, substitute e for ξ and ω_1 for ω, and we get

$$e = r \sin(\omega_1 + \delta).$$

Next follows, as shown in Fig. 12, for the greatest distance of the valve from its central position

$$r = e + a + k.$$

By combining these two equations and using at the same time the formula for δ, we get after simple reduction

$$2\,r = \frac{a + k}{\sin^2 \frac{\omega_1 + \gamma}{4}}.$$

From this equation the eccentricity $O\,P = r$ may readily be calculated (or may be constructed according to its direction).

Now by describing the valve circle on O P, Fig. 12, the two points of intersection, V_1 and V_4, upon the crank lines $O\,R_1$ and $O\,R_4$ are obtained, the lengths $O\,V_1$ and $O\,V_4$ representing the outside lap e and $V_0\,P_0$ the required outside lead.

Finally, we may assume the inside lap and thus complete the diagram.

Problem 4.—"Let there be given the degree of expansion $\epsilon = \frac{s_1}{s}$, the outside lead $V_0\,P_0 = v$ (Fig. 12), the width of ports $V\,M = a$, and $M\,P = k$ the greatest distance to which the outer edge of the valve passes beyond the inner edge of the steam-port.

"Required, the radius of eccentricity r, the angle of advance δ, the outside lap e and the angle of lead γ." *

Solution.—This problem is most readily solved by calculating the eccentricity r, and then determining the other unknown quantities by construction.

The formula according to which r is calculated, may be found as follows:—

For the beginning of the stroke of the piston we must, in the general equation of the travel of the valve $\xi = r \sin(\omega + \delta)$, substitute 0 for ω and (Fig. 12) $e + v$ for ξ, and accordingly get

$$e + v = r \sin \delta.$$

Next we determine, either by calculation or construction, the angular movement ω of the crank up to the beginning of the expansion, and substituting this value and $\xi = e$ in the general formula, we obtain

$$e = r \sin(\omega_1 + \delta).$$

Finally, according to the diagram (Fig. 12) the greatest travel of the valve is

$$r = e + a + k.$$

From these three equations we have to find the three unknown quantities r, δ, and e. Let the value for e, obtained from the last equation, be substituted in the two first, then

$$r(1 - \sin \delta) = a + k - v, \qquad (\alpha)$$

and

$$r[1 - \sin(\omega_1 + \delta)] = a + k. \qquad (\beta)$$

Now in order to find r from these two equations, δ must disappear. Use can be made here of the known formula

$$1 - \sin x = 2 \sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right),$$

and thus there can be obtained from equation (α)

$$\sin\left(\frac{\pi}{4} - \frac{\delta}{2}\right) = \sqrt{\frac{a + k - v}{2r}},$$

* The above-given problem was first solved by Redtenbacher ('Gesetze des Locomotiv-baues,' p. 107) by calculation; later the engineer Hermann gave in the 'Zeitschrift des österreichischen Ingenieur-Vereines,' 1859, vol. ix., p. 81, and Prof. Grashof in the 'Zeitschrift des Vereines deutscher Ingenieure,' 1859, vol. iii., p. 294, a real graphical solution of the same problem.

and

$$\cos\left(\frac{\pi}{4} - \frac{\delta}{2}\right) = \sqrt{\frac{2\,r - (a + k - v)}{2\,r}}.$$

But the equation (β) gives

$$\sin\left(\frac{\pi}{4} - \frac{\delta}{2} - \frac{\omega_1}{2}\right) = -\sqrt{\frac{a + k}{2\,r}},*$$

or

$$\sin\left(\frac{\pi}{4} - \frac{\delta}{2}\right)\cos\frac{\omega_1}{2} - \cos\left(\frac{\pi}{4} - \frac{\delta}{2}\right)\sin\frac{\omega_1}{2} = -\sqrt{\frac{a + k}{2\,r}}.$$

By using in the formula the two expressions above given, and by considering that here, as in the previous problems,

$$\sin\frac{\omega_1}{2} = \sqrt{\frac{s_1}{s}} = \sqrt{\epsilon},$$

and therefore

$$\cos\frac{\omega_1}{2} = \sqrt{1 - \epsilon},$$

we get after reduction

$$\sqrt{1 - \epsilon}\,\sqrt{a + k - v} - \sqrt{\epsilon}\,\sqrt{2\,r - (a + k - v)} = -\sqrt{a + k},$$

whence follows

$$r = \frac{2\,(a + k) - v + 2\,\sqrt{(a + k)\,(a + k - v)\,(1 - \epsilon)}}{2\,\epsilon}.$$

We can now calculate by this formula the eccentricity r, and as for the rest, use the system of construction.

After drawing the two axes O X and O Y (Fig. 12), describe from O, with the radius O X = O P = r, a circle (shown in the figure by dotted lines), mark from X towards O the value $a + k$, then the outside lap O $V_0 = e$ is obtained, with which we can describe from O the lap circle.

Next lay off from V_0 towards X the outside lead $V_0 P_0 = v$, and erect at P_0 a perpendicular P_0 P reaching the circle described with r at P; then O P will be the real direction of the eccentricity r, P O Y will be the required angle of advance δ, and the circle described on

* [In simple valve-gear $\frac{\delta + \omega_1}{2}$ is always greater than $\frac{\pi}{4}$, hence $\sqrt{\frac{a + k}{2\,r}}$ must be preceded by a negative sign.—Ed.]

O P from the centre C will be the valve circle. The points V_1 and V_4 give the positions of the crank O R_1 and O R_4 for the beginning of the expansion and the admission of the steam, and the angle X O R_4 is the required angle of lead. The completion of the diagram requires only that the inside lap be assumed.

If, for example, we take the degree of expansion $\epsilon = 0 \cdot 8$, the outside lead $v = 0 \cdot 006^m$ ($\cdot 23''$), the width of ports $a = 0 \cdot 030^m$, and the distance to which the valve at its greatest travel passes beyond the inner edge of the entrance-port $k = 0 \cdot 006^m$ ($\cdot 23''$), then the above formula gives for r

$$r = 0 \cdot 060^m \ (2 \cdot 36''),$$

and from the given construction follows

$$e = 0 \cdot 024 \quad \text{and} \quad \delta = 30^\circ,$$

which values may also be obtained through calculation from the formulas

$$e = r - (a + k)$$

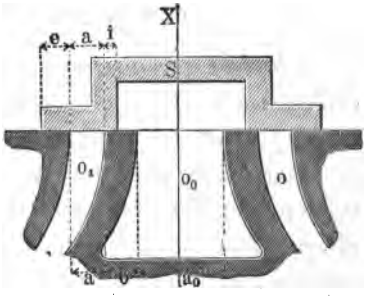
and

$$\sin \delta = \frac{e + v}{r}.$$

Practical Notes on the Designing of Valve-gears.

In executed valve-gears it is generally found that the eccentricity r = from $0 \cdot 050^m$ ($1 \cdot 96''$) to $0 \cdot 080^m$ ($3 \cdot 14''$), that the angle of advance = from 10° to 30°, and that the outside lead = from $0 \cdot 003^m$ ($\cdot 118''$) to $0 \cdot 006^m$ ($\cdot 23''$).

Fig. 13.



The width of the steam-ports, measured in the direction of the motion of the valve, is generally from $0 \cdot 030^m$ to $0 \cdot 050^m$ ($1 \cdot 18''$ to $1 \cdot 97''$).

Finally, after all these dimensions are fixed, there only remains to be determined the width b of the bridges and the width o_0 of the exhaust-port (Fig. 13). Of course the bridge must be wider than

the length M P (Fig. 12), because otherwise the valve would open the exhaust-port and allow fresh steam to escape into the air.

As the greatest distance of the valve from its central position is given by the eccentricity r, it will be seen that

$$e + a + b \text{ must be greater than } r;$$

therefore

$$b \text{ is greater than } r - (e + a).$$

Useful proportions for this width b may be obtained by applying the following empirical formula

$$b = 10 + 0{\cdot}5\, a \text{ in millimetres } [b = 0{\cdot}4 + 0{\cdot}5\, a \text{ in inches}],$$

in which the width a of the entrance-port is to be taken in millimetres [inches].

Making also the width a_0 of the exhaust-port

$$a_0 = r + a + i - b,$$

the valve itself at its farthest travel will only reduce the exhaust-port as much as the width of the entrance-port. For the case observed in the above problem 2 we had as an example

$$r = 60^{\text{mm}},\ e = 24^{\text{mm}},\ i = 7^{\text{mm}}\ [r = 2{\cdot}36'',\ e = 0{\cdot}94'',\ i = 0{\cdot}272''],$$

and taking the width of the entrance-port $a = 30^{\text{mm}}$ ($1{\cdot}18''$), we get, according to the preceding formula, the width b of the bridges

$$b = 10 + 0{\cdot}5 \times 30 = 25^{\text{mm}}\ ({\cdot}985''),$$

and the width of the exhaust-port

$$a_0 = 60 + 30 + 7 - 25 = 72^{\text{mm}}\ (2{\cdot}808'').*$$

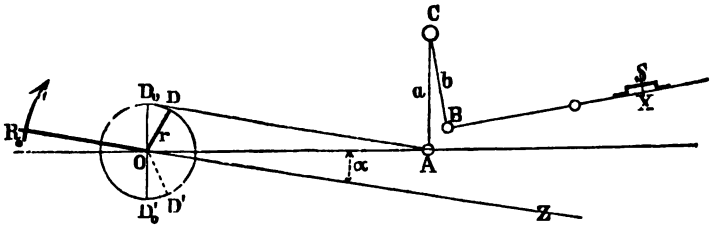
A further question, and one moreover which often presents great difficulties to the beginner, is that respecting the angle which the centre line of the eccentric should make with that of the crank, or in other words, how the eccentric is to be keyed to the axle with reference to the position of the crank when the angle of advance is given. The rules which here follow are very simple, and long experiments on models are quite unnecessary for obtaining a clear understanding of this question.

* [See note and diagram at end of Section I.—Ed.]

In practice, almost all the cases which occur may be divided into two classes, which are represented in Figs. 14 and 15. Thus the eccentric may act through the eccentric-rod either upon the bell-crank lever A C B (Fig. 14), or upon the double lever A C B (Fig. 15).

Let us first consider the case shown in Fig. 14. There is always given the position and direction of the valve-face B X, as well as the position of the centre O of the axle and the direction O Z of the axis of the steam-cylinder.

FIG. 14.



Choose the direction O A, in which the end A of the eccentric-rod shall swing back and forth; draw the valve S in its central position and choose two points B and A, the first in the direction of the valve-face, the other in the chosen direction O A; erect on the lines representing these directions the perpendiculars B C and A C; then will C be the position of the fulcrum of the bell-crank lever A C B, and this lever will occupy the position in which it stands when the valve is in its central position. Draw now from the centre O the line O D_0 at right angles to the chosen direction O A, and make O D_0 equal to the eccentricity r; then the connecting line D_0 A gives the length of the eccentric-rod. Produce O Z, the axis of the cylinder, past O, and lay off upon this the crank O R_0 as occupying one of its dead points; if we now draw the line O D = r so that it forms with the line O D_0 the angle D_0 O D = δ, equal to the angle of advance, and suppose the crank to move in the direction of the arrow, then R_0 O D is the angle by which the centre line of the eccentric must precede the crank. The difference between the two directions will be, if the axis of the cylinder O Z differs from the chosen direction O A by the angle a,

$$\text{angle } R_0 O D = 90^\circ + \delta - a.$$

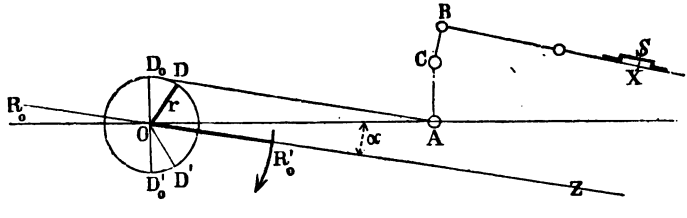
If, however, the engine runs in the direction opposite to the arrow, the perpendicular O D_0 must be drawn downwards, and the centre line of the eccentric r must be put in the position O D′ (shown in the figure in dotted lines), so that the angle D_0' O D′ becomes equal to δ and the centre line of the eccentric *precedes* the crank by the angle

$$R_0 O D' = 90° + \delta + \alpha.$$

If in the present case the valve-face B X were parallel to the chosen direction O A, the joint B of the valve-rod would be in the line A C, and the bell-crank lever would become simply a one-armed lever. But in most cases the direction B X coincides with the direction OA, when BC = AC, and the lever is omitted altogether, a separate guide for the end of the eccentric-rod being no longer required.

The second case, as represented in Fig. 15, may be treated in a similar manner. Let the centre of the axle O, the direction B X of the valve-face, and O Z of the axis of the cylinder, again be given.

Fig. 15.



Choose as before the direction O A in which the end-point A of the eccentric-rod shall move: draw the valve S in the central position, and B C perpendicular to B X, and A C perpendicular to O A; A C B is then the double lever with the fulcrum C, which transfers the eccentric motion to the valve, the lever being in its central position. By now drawing O $D_0 = r$ perpendicular to O A, the connecting line D_0 A again gives the length of the eccentric-rod. Continue as in the first case the line O Z; draw the crank O R_0 towards the dead point farthest from cylinder, and draw the eccentricity O D = r, so that it forms with O D_0 the angle D_0 O D = δ, the

angle of advance, the crank being supposed to move in the direction indicated by the arrow. If, however, the crank moves in the opposite direction, draw the line O D′ = r, as in the preceding case.

But in the present instance the proper position of the crank is 180° from that just drawn, because the direction of motion of the valve is reversed by the double lever; the crank must therefore, for the given position of the eccentric, be placed in *the direction* O R_0', and it will be found that the eccentricity O D = r, when the crank moves in the direction indicated by the arrow, follows *behind* the crank at the fixed angle

$$R_0' O D = 90° - \delta + \alpha.$$

But when the crank revolves in the opposite direction, the angle will be

$$R_0' O D' = 90° - \delta - \alpha.$$

If we call the arms A C and B C in both cases a and b, the movement of the valve will be

$$\xi = \frac{b}{a} r \sin (\omega + \delta).$$

In drawing out the diagram, therefore, the diameter of the valve circle is to be taken as $\frac{b}{a} r$, but nothing else is to be altered in the rules for designing and using the diagram. In all our investigations we have considered the simpler and more frequently occurring case, and tacitly assumed that $a = b$, and that the direction of the valve-face coincided with the direction O A, or in other words, that the end of the eccentric-rod acted directly on the end of the valve-rod.

CHAPTER II.

On the "Missing Quantity" in the Formula for the Simple Valve-motion.

Equation (I^b) gave for the movement of the valve

$$\xi = r \sin \delta \cos \omega + r \cos \delta \sin \omega + \frac{r^2}{2\,l} \sin \omega \sin (2\,\delta + \omega).$$

In the preceding investigations we neglected the third quantity, because in most cases it is very small; the first two terms then represented the polar equation of two circles, which explained the whole distribution of the steam; moreover, it then appeared that the valve moved symmetrically back and forth on both sides of a point which was at a fixed distance from the centre of the axle.

In reality however this swinging motion is not so regular, for the quantity

$$\frac{r^2}{2\,l} \sin \omega \sin (2\,\delta + \omega)$$

cannot under the usual arrangements be made to disappear altogether. But generally the length l of the eccentric-rod is very large in proportion to the eccentricity r, and the disadvantageous influence of the *missing quantity* nearly disappears in practice. Nevertheless it seems important to us to examine the influence of this quantity more closely, as we can get from it a practical rule, and as the exact movement of the valve will be thus brought before our eyes.

We find (see p. 47) for the distance of the central position B of the valve from the centre of the axle O, when the crank has moved through the angle ω (Fig. 16),

$$\text{O B} = r \sin (\omega + \delta) + l + l_1 - \frac{r^2 \cos^2 (\omega + \delta)}{2\,l}.$$

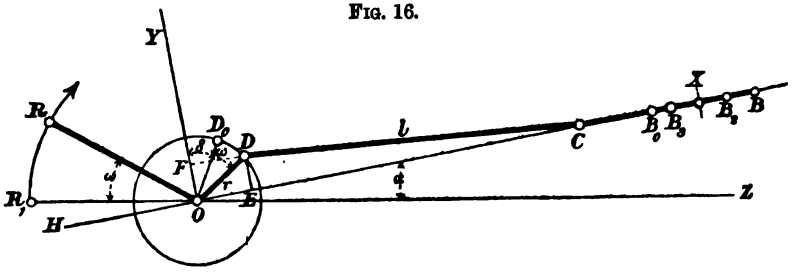
If we substitute $180° + \omega$ for ω in this equation, that is, suppose the crank to have turned through 180°, we find the distance of the centre of the valve B_0 to be

$$\text{O B}_0 = r \sin (180° + \omega + \delta) + l + l_1 - \frac{r^2 \cos^2 (180° + \omega + \delta)}{2\,l}.$$

But the average of both values gives the distance of the middle point of the line $B B_0$ from O; calling this distance $O X_1$, we get, after some reductions,

$$O X_1 = l + l_1 - \frac{r^2 \cos^2 (\omega + \delta)}{2 l}. \qquad (11)$$

FIG. 16.



Strictly speaking X_1 is nothing but the centre of motion, and $O X_1$ is therefore its distance from the centre of the axle. As ω occurs in the corresponding formula, it is evident that $O X_1$ is variable, and *therefore that the position of the centre of motion is likewise variable.*

We found, equation (6), the distance of the centre of motion to be

$$O X = l + l_1 - \frac{r^2 \cos^2 \delta}{2 l},$$

by ascertaining the positions of the valve for $\omega = 0$ and $\omega = 180°$, and then taking the point mid-way between these as the fixed centre of motion. We based this upon the following method, pursued in practice when *setting* or adjusting a valve: The crank is placed at the dead point, and the lead measured, i. e. the opening of the entrance-port for this position of the crank is measured; the crank is next placed on the second dead point, and the corresponding lead measured, and the valve-spindle is then shortened or lengthened till the lead on both sides is equal. This is also, as will be shown, the most correct way, and coincides perfectly with our method of determining the position of the point X.

When the "missing quantity" was neglected, the centre obtained was X; but when this quantity is taken into consideration, the actual

position of the centre of motion X_1 will be found to be different from X, and the amount by which it is thus shifted is given by

$$z = O X_1 - O X = \frac{r^2}{2\,l}\left[\cos^2\delta - \cos^2(\omega + \delta)\right],$$

or, after a few reductions,

$$z = \frac{r^2}{2\,l}\sin\omega\sin(2\,\delta + \omega), \tag{12}$$

and that is *nothing but the missing quantity itself.* An examination of this equation therefore will give a complete explanation of the movement of the variable centre of motion X_1 with respect to the fixed centre X.

Both coincide, or $z = 0$, when $\omega = 0°$, $(180° - 2\,\delta)$, $180°$ and $(360° - 2\,\delta)$, i. e. when the crank stands *on the dead points* or as much as $2\,\delta$ *before them.*

The "missing quantity" is a maximum, or the difference z between the positions of the two points X and X_1 is greatest, when

$$\sin\omega\sin(2\,\delta + \omega) = \text{maximum},$$

or according to the calculus, when

$$\sin 2\,(\delta + \omega) = 0.$$

This again gives four angles, viz.:

$$\omega = (90° - \delta);\ (180° - \delta);\ (270° - \delta);\ \text{and}\ (360° - \delta).$$

The two angles $90° - \delta$ and $270° - \delta$ give the greatest positive value of z:

$$z_{max} = +\frac{r^2}{2\,l}\cos^2\delta. \tag{13}$$

The two other angles $(180° - \delta)$ and $(360° - \delta)$ give

$$z_{max} = -\frac{r^2}{2\,l}\sin^2\delta. \tag{14}$$

But the diagram (Fig. 2, Pl. I.) shows that just at the angles $90° - \delta$ and $270° - \delta$, the largest opening of the ports takes place, and the valve, being at the extremities of its stroke, travels a small distance beyond the edge of the port; now whether, in consequence

of the missing quantity, the movement of the valve is more or less than r by $\frac{r^2}{2l}\cos^2\delta$ is a matter of no importance whatever. The greatest difference between the positions of the points X and X_1 occurs therefore at a place where it cannot influence the distribution of the steam.

The circumstances are similarly favourable in the cases where the second maximum difference occurs.

This second difference occurs when the angles ω are $180° - \delta$ and $360° - \delta$; but according to the diagram, these angles represent those positions of the crank at which the valve is just at the middle of its stroke, that is, when both ports are closed. The effect of the above difference is, that the valve, at these moments, has not yet quite reached the position it otherwise would have, being still distant from it to the small extent $\frac{r^2}{2l}\sin^2\delta$; but as the ports are closed at these positions by the lap, the disturbance in this case also occurs at a time when it is least felt.*

We have now seen that the greatest differences from the results given by our diagram, take place at points where they are without unfavourable influence upon the distribution of the steam; the reason for this principally lies in the correct fixing of the centre of motion, in other words, in the manner of adjusting or setting the valves explained above. From this follows the practical rule, always *to adjust the valves so that the lead is equal on both sides.*

In order to get a complete view of the influence of the missing quantity upon the entire distribution of the steam, the exact values of ξ must be calculated for different values of ω, according to equation (I^a), and the curve be drawn to polar co-ordinates. Instead of the two circles, a curve is obtained which likewise has the shape of an 8; this curve then takes the place of the two valve circles, and in connection with the other circles explains exactly,

* If the laps are very small, perhaps *nil*, as is often the case with the inside lap, then of course an unfavourable distribution of the steam may be expected in consequence of the effect of the missing quantity. It is therefore advisable, when great eccentricities and short eccentric-rods are employed, to make the laps larger than usual.

according to the instructions given with regard to Fig. 2, Pl. I., the distribution of the steam.

We do not enter upon this question more minutely, as it is of no further use. In most cases the ratio $\frac{r}{l}$ is quite small, and then there exists no reason for regarding the "missing quantity," and thus making the investigation more difficult. Moreover the irregularities due to the *piston motion,* which were emphasized on p. 7, have a greater hurtful influence on the distribution of the steam than the "missing quantity," and in more exact investigations the former must be first taken into account.

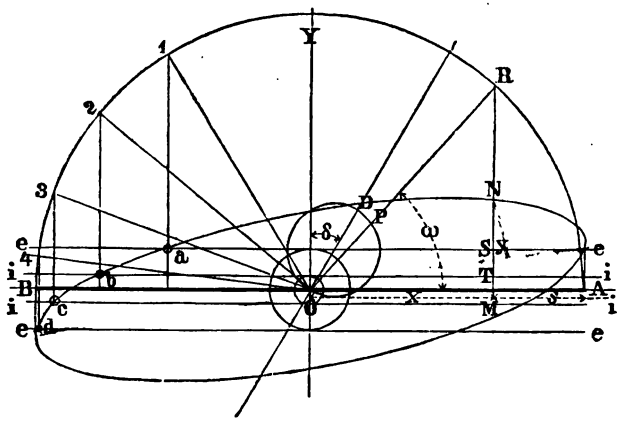
Representation of the Distribution of Steam by Means of Valve Ellipses.

The simple diagram for the movement of the slide-valve already described and examined, gives for any crank angle ω the distance of the valve from its central position, or, as we may say, it represents *the valve movement ξ as a function of the angular movement ω.* If the angle ω is given, it is easy to fix the position which the piston at the same moment occupies in the cylinder. It may be now desirable to show directly, graphically, the relation between the movement of the valve and the stroke of the piston; and for such a purpose, a system of rectangular co-ordinates is to be adopted, in which the stroke of the piston is taken as abscissa and the travel of the valve as ordinate; the different points obtained in this manner for different angular movements are connected by a curve.

We will now consider more closely this mode of representation, which was formerly generally used and which is still applied by a few authors. Let (Fig. 17) A B be the full stroke of the piston, drawn to a reduced scale, and on A B as a diameter and from O, with the radius O A = O B = O R = R, let there be described a circle. Let O R be supposed to represent the crank, and let it have moved from the position O A (the dead point) as much as the angle A O R = ω, then the perpendicular R M gives at once in the part A M = s of the line A B, the *distance of the piston from the end of its stroke.* This line may be taken as an abscissa, when A is taken as the origin of

co-ordinates, and when, as we shall for simplicity suppose here, the connecting-rod is taken as infinitely long. Let us now draw to the same or any other scale, the ordinate M N representing the movement ξ of the valve, corresponding to this position of the crank. This

FIG. 17.



movement of the valve may be either calculated by the formula $\xi = r \sin(\omega + \delta)$, or it may be obtained in a graphical manner by our polar diagram. In the latter case, draw from O the perpendicular O Y, and draw next the eccentricity O D = r in such direction that the angle Y O D becomes equal to δ, the angle of advance. The radius vector O P, falling in the direction of the crank O R, and belonging to a circle described on O D, thus gives at once the required ordinate M N.

Fixing in this way a series of points like N, and connecting them by a line, we obtain a closed curve which, as may easily be proved, is an ellipse, and which is called the valve ellipse. If we now shift the origin of co-ordinates to the centre O, and take O M = x as the abscissa of the point N, we at once find (this being exact only for connecting-rods of infinite length)

$$x = R \cos \omega, \quad = a \cos \theta$$

while the ordinate, which for the present we will call y, is

$$y = \xi = r \sin(\omega + \delta). \quad = b \sin \theta$$

Eliminating from both equations the angle ω, we get, after a few reductions,

$$R^2 y^2 - 2 R r y x \sin \delta + r^2 x^2 - R^2 r^2 \cos^2 \delta = 0;$$

and that is, as may easily be proved, the equation of an ellipse referred to rectangular axes with the centre as an origin, but in which the axes of the co-ordinates do not coincide with the principal axes.* The ordinates of the different points of that part of the ellipse situated above A B represent the movements of the valve towards the one, and the ordinates of the points below A B the movements towards the other side of the centre of motion. By drawing the parallel lines *e e* and *e e* at a distance equal to the outside lap *e* above and below A B, and also the parallel lines *i i* and *i i* at a distance *i* equal to the inside lap, the distribution of the steam is given, and it will be found that the opening of the entrance-port for the position M of the piston during the stroke of the piston from A to B is given by the part N S of the ordinate M N, and the port opening for the exhaust by the part N T of the same ordinate. Moreover, it is easy to understand that the points *a*, ~~c~~ b indicate respectively the position of the piston for the beginning of the expansion and of the exhaust in the cylinder end nearest the crank-shaft, and the points *b*, *d*, the beginning of the compression and admission of the steam in the cylinder end farthest from crank-shaft.

* Calling the angle which the major axis of this ellipse makes with the axis of abscissa A B, ϕ; then starting with the equation of the ellipse, $a^2 + b^2 \sin^2 \varphi$

$$a^2 y_1^2 + b^2 x_1^2 = a^2 b^2,$$

referred to its principal axes, and transforming it so that the ellipse is referred to the axes O Y and O A, we get an equation similar to the one just developed; equating the corresponding coefficients in the two equations, we have

$$tg.\, 2\phi = \frac{2 R r \sin \delta}{R^2 - r^2};$$

for the semi-transverse axis *a*,

$$a^2 = \frac{R^2 + r^2 + \sqrt{(R^2 + r^2)^2 - 4 R^2 r^2 \cos^2 \delta}}{2};$$

and for the semi-conjugate axis *b*,

$$b^2 = \frac{R^2 + r^2 - \sqrt{(R^2 + r^2)^2 - 4 R^2 r^2 \cos^2 \delta}}{2}.$$

By producing the perpendiculars to A B that pass through these points of the curve, till they meet the crank-pin circle, we obtain for the four principal points in the distribution of the steam, the positions of the crank O 1, O 2, O 3, and O 4. We, however, found these positions more simply in our polar diagram through the points in which the valve circles and the two lap circles crossed each other. In fact, this method of representing the movement of the valve by an ellipse does not give a better view of the matter than our former diagram, and is not to be recommended for practical application, on account of the greater difficulty of construction.

For those valve-gears in which the eccentricity and angle of advance are variable, this method is without value; for each separate case would require the construction of a special ellipse, a work which requires much time and does not give the necessary exactness in the measurements; besides, the figure would be covered with so many lines and curves that it would be difficult to understand.*

Reuleaux's Diagram.

A diagram different from that above described, and which with equal facility answers all questions relating to the distribution of the steam and the influence of different dimensions in a simple valve-gear, has been invented by Professor Reuleaux. In the following description of this diagram by Reuleaux, the same designations are employed, so as to facilitate comparison with the other diagram.

The travel of the valve has been shown, according to previous formulas, to be

$$\xi = r \sin \delta \cos \omega + r \cos \delta \sin \omega,$$

or

$$\xi = r \sin (\delta + \omega).$$

* [This elliptical diagram is best known in English and American engineering circles as the *motion-curve;* once constructed, it gives directly the relation between piston positions and valve positions, which relation it is the object of valve-gear investigation to determine; it will, therefore, continue to be used as a representation of the final, desired, result; for constructing this motion-curve we may use Professor Zeuner's simple diagram for obtaining the valve movements (ordinates) and the tables on pp. 8 and 9 for determining the piston positions (abscissas).—Ed.]

The opening of port for admission when the crank angle is ω, is

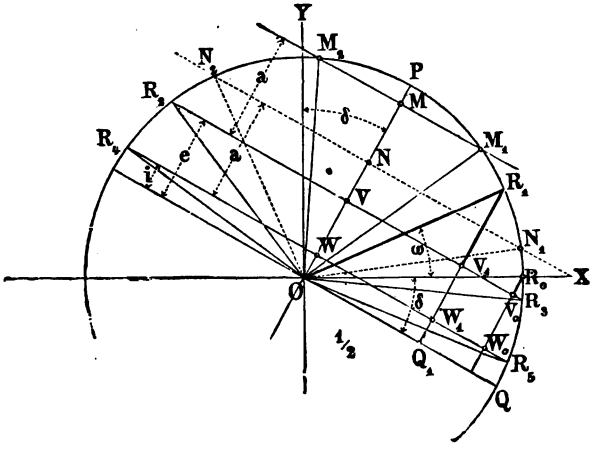
$$\xi - e = r \sin(\delta + \omega) - e,$$

and the opening of the port for the exhaust, is

$$\xi - i = r \sin(\delta + \omega) - i.$$

Now in order to save calculations and to obtain by diagram a complete insight into the matter, we proceed according to Reuleaux

Fig. 18.



as follows: Draw the two axes O X and O Y at right angles to each other (Fig. 18), take O as the centre of the axle, and draw the lines O P and O Q so that they form with O Y and O X the angles

$$YOP = XOQ = \delta,$$

i. e. equal to the angle of advance. Now describe a circle from O as a centre, with the eccentricity O R $= r$ as a radius, and suppose the crank to be in the position O X on one of its dead points. Next draw the line $R_4 R_5$ parallel to the line O Q at the distance O W $=$ $i =$ the inside lap; and also the line $R_2 R_3$ at the distance O V $= e$ $=$ the outside lap; and finally the line $M_1 M_2$ at the distance V M $= a =$ the width of the entrance-port, from $R_2 R_3$, and the diagram is completed.

Suppose the crank to have turned through the angle X O $R_1 = \omega$,

and therefore to be in the position $O R_1$, and drop from R_1 the perpendicular $R_1 Q_1$ on to $O Q$, then we have

$$R_1 Q_1 = r \sin (\omega + \delta)$$

the movement of the valve, as well as

$$R_1 V_1 = R_1 Q_1 - Q_1 V_1 = r \sin (\omega + \delta) - e$$

the opening of the port for admission, and finally

$$R_1 W_1 = R_1 Q_1 - Q_1 W_1 = r \sin (\omega + \delta) - i$$

the opening of the port for exhaust for this particular position of the crank.

If the crank stands on the dead point, for instance in the position $O R_0$, the opening for the admission of the steam is $R_0 V_0$, and that for the exhaust is $R_0 W_0$; the former value is therefore the outside, and the latter the inside, lead.

The correctness of the following will be very easily understood. At the position $O R_3$ of the crank, when it stands at the angle $R_3 O R_0$ *before* the dead point, the admission of the steam begins in the cylinder end nearest the crank-shaft. At the position $O R_5$ the exhaust begins in the cylinder end farthest from crank-shaft.

At the position $O M_1$ of the crank, the entrance-port is just fully open, and it remains fully uncovered until the position $O M_2$ is reached by the crank.

At $O R_2$ the expansion begins, at $O R_4$ the compression, &c., &c.

If the crank occupies the position $O Q$, the valve is at its central position, and when the crank is at the position $O P$, the valve is most remote from its central position, and $M P$ represents the distance to which the outer edge of the valve has passed beyond the inner edge of the port.

If it is also required to know at which positions of the crank the port at the other end of the cylinder is just fully open for the exhaust, or just beginning to shut, draw at the distance $W N = a$ the line $N_1 N_2$ parallel to $O Q$, and then $O N_1$ and $O N_2$ are the required positions.

In the other semicircle the procedure is of course the same, and all the principal positions of the crank have only to be produced through O till they meet the circumference of the circle.

As the values of r, e, a, and i are generally drawn full-size, all the other measurements are also obtained to the same scale.

In the diagram (Fig. 18), which is drawn half-size, the data are the same as in problem 2.

This diagram gives for the simple valve-gear the principal positions of the crank as readily as the circle diagram; and it gives just as simply and clearly the proportions for valve-gears with separate expansion valves. It only loses in clearness if it is applied to those valve-gears in which the eccentricity as well as the angle of advance is variable; for such valve-gears, of which we shall examine a whole series farther on, the circle diagram is to be preferred.

Deprez's Diagram.

Deprez's diagram was recently made known in an article by *Combes*,* and represented by the latter as being as simple and clear as the diagram which is the subject of the present work.

We will now briefly describe Deprez's diagram, but will say in advance that, when applied to complicated link-motions it undoubtedly loses in clearness, and can only be compared with other diagrams when applied to valve-gears with one valve and fixed expansion.

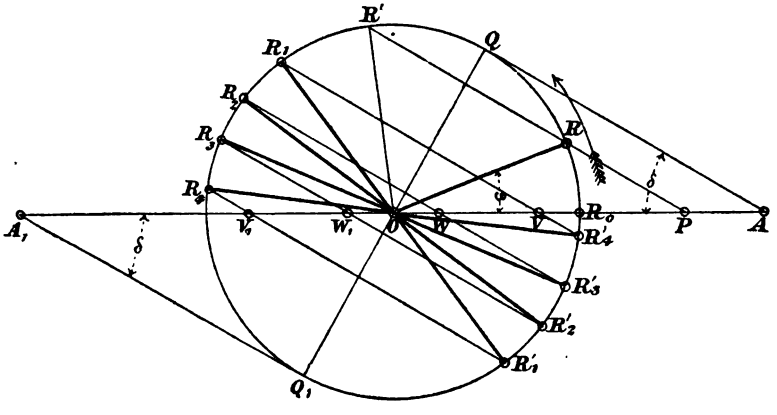
Let A_1 A (Fig. 19) represent the direction of stroke of the end of the eccentric-rod, and O the centre of the engine-shaft. Lay off to the right and left of O the distance O A = O A_1 equal to the *eccentricity r*, and draw the lines A Q and A_1 Q_1 so that they make, with the direction A_1 A, angles O A Q and O A_1 Q, each equal to δ, i. e. equal to the given *angle of advance;* now let fall from the centre O, on A Q and A_1 Q_1, the perpendiculars O Q and O Q_1, and, with O as a centre and O Q as a radius, describe the circle R_0 R Q R_1;

* Combes, 'Bulletin de la Société d'Encouragement,' &c., 1868, p. 526.

in accordance with this construction the radius of the circle is $O\,Q = r \sin \delta$, and the lines $A\,Q$ and $A_1\,Q_1$ are tangent to the circle which we may also regard as the crank-pin circle.

Now if we suppose the crank to have moved in the direction of the arrow through an angle $R\,O\,R_0 = \omega$, estimated from the dead-

FIG. 19.



point position $O\,R_0$, and draw through the point R a parallel $R\,P$ to the tangent $A\,Q$ till it intersects at P the direction of the stroke $A_1\,A$, then will the distance $O\,P$ be the distance of the valve from its central position, i. e. $O\,P$ will represent the valve movement ξ for the corresponding crank position.

In accordance with the construction, we have in the triangle $O\,R\,P$:

$$R\,O\,P = \omega \quad \text{and} \quad O\,P\,R = \delta,$$

hence

$$O\,R\,P = 180^\circ - (\delta + \omega),$$

and it follows that

$$O\,P : O\,R = \sin\,[180^\circ - (\delta + \omega)] : \sin \delta,$$

and from this we obtain, since $O\,R = r \sin \delta$,

$$O\,P = \xi = r \sin\,(\delta + \omega).$$

Therefore, to determine the valve movement from Deprez's diagram, we have this rule:—Project the given crank position $O R = r \sin \delta$ obliquely, i. e. parallel to the tangent $A Q$, on to the valve direction $A A_1$; the projection $O P$ is the valve movement.

The inverse process of obtaining the crank position corresponding to the valve movement is effected by drawing through P a parallel to the tangent $A Q$, and finding the intersections R and R′; the crank positions $O R$ and $O R'$ are those which have the same valve movement $O P$; it follows from this that the valve will be at the greatest distance $O A = O A_1 = r$ from its central position when the crank is in the positions $O Q$ and $O Q_1$.

To find the most important crank positions connected with the distribution of the steam, lay off on $A A_1$ to the right and left of O the distances $O V = O V_1 = e$ equal to the *outside* lead, and the distances $O W = O W_1 = i$ equal to the *inside* lead, and through the points V, W, W_1 and V_1 draw parallels to $A Q$, and note their intersections R_1, R_2, &c., with the circle. We then have the principal crank positions for the crank motion through the upper semicircle; the crank stands at $O R_1$ when expansion begins in the cylinder end nearest crank-shaft, and at $O R_2$ when compression begins in the cylinder end farthest from shaft; at $O R_3$ when steam begins to exhaust from the cylinder end nearest shaft; at $O R_4$ when steam is admitted to cylinder end farthest from crank. For the crank motion through the lower semicircle we have the corresponding crank positions $O R'_1$, $O R'_2$, $O R'_3$ and $O R'_4$.

When the diagram is drawn full-size the *outside* lead is given directly by the distance $V R_0$ and the *inside* lead by the distance $W R_0$.

It is unnecessary to enter into the other questions connected with simple valve-gears, for they have been fully discussed in connection with the polar diagram, and their transference to Deprez's diagram is not accompanied by any special difficulty.

MÜLLER'S DIAGRAM.

Another graphical method of ascertaining the different positions of the valve relatively to the different positions of the crank has been given by Professor Müller.*

Müller's diagram, which distinguishes itself from the above-given diagrams principally by representing the valve movement in a mathematically exact manner, may be described as follows :—

Let O X (Fig. 4, Pl. I.) again be the direction of the valve-face, and let O Y be perpendicular to this direction. If the crank is on the dead point, the eccentricity r takes the position O D; D O Y being the angle of advance δ. Next let DB_2 be the eccentric-rod, the length of which we shall call l, and B_2 may represent directly the centre of the valve. If the crank is now turned through the angle ω, then the centre line of the eccentric r is shifted from the position O D into that marked O D′, and the valve centre comes to B_0. The distance OB_0 has been already determined by formula (p. 12); but this distance may also be obtained by geometrical means, as follows: Instead of regarding the valve-face O X as fixed, and O D as changing its position to OD_1, as is the case in reality, we regard the eccentricity O D as fixed, and turn the valve-face O X in the opposite direction through the angle $XOX' = \omega$ (direction of the arrow). From D as a centre, with the eccentric-rod $BD = l$ as a radius, describe an arc cutting O X′ at B, the length O B gives at once the distance of the valve centre from the centre of the axle for the angle ω, for $OB = OB_0$. Hence follows the simple rule:—In order to fix the distance of the valve centre B from the centre O of the axle, describe from the point D a circle with the length of the eccentric-rod $DB_2 = DB = l$ as a radius; the radial lines drawn to that circle from O then directly represent the required distances; and the angle which any radial line O B forms with the direction O X, gives at the same time the angle ω which the crank at this position of the valve makes with the axis of the cylinder.

But the valve has to swing to each side of a fixed centre. If we

* Invitation-paper of the Royal Polytechnical School at Stuttgart. 1859. Civil-ingenieur, vol. vii. p. 347.

adjust the valve so as to have equal lead at each end of cylinder, i. e. set the valve so that it stands at equal distances from the centre of oscillation when the crank passes through one or the other dead point, then in order to determine the movement of the valve, we must proceed in the following manner:—

Bisect the distance $B_2 B_3$, i. e. the distance between the points in which the circle cuts the direction O X, in the point O_1, and describe from the centre O of the axle, with the radius $O M = O_1 B_2 = O_1 B_3$, a second circle, then the length M B gives directly for the crank angle ω the movement ξ of the valve, or the distance travelled by the valve from its central position. Producing O B past O, we get, in the same manner, the length M′ B′ as the movement of the valve for the angle $180° + \omega$; the lengths $M_2 B_2$ and $M_3 B_3$ give the movement of the valve for $\omega = 0$ and $\omega = 180°$, i. e. for the moments at which the crank passes through the one or the other dead point. It is also easy to understand that O M_0 and O M'_0 will represent the positions occupied by the crank when the valve is just at the centre of its travel.

The openings of the port for admission and exhaust may also be easily obtained for any position of the crank. Laying off upon the line O B the distance $M P = M' P' = e$, i. e. equal to the outside lap, and the length $M N = M' N' = i$ equal to the inside lap, and describing from O with the lengths O P, O N, O P′, O N′ a series of circles, B P gives the opening of the port for the admission and B N the opening of the port for the exhaust when the crank angle is equal to ω; and in the same manner the lengths B′ P′ and B′ N′ give the same openings for the angle $180° + \omega$.

The distances $B_2 P_2$ and $B_3 P_3$ represent the openings of the ports for admission, and $B_2 N_2$ and $B_3 N_3$ the openings of the ports for the exhaust, when the crank stands on one or the other dead point, the piston being then at the end of its stroke; the former distances represent therefore the outside lead, and the latter the inside lead. It will be seen finally, considering the statements made previously in constructing the diagram, that for the forward and return stroke of the piston respectively O R_1 and O R'_1 in Fig. 4, Pl. I., represent the positions of the crank at the beginning of the expansion;

$O R_2$ and $O R'_2$ those at the beginning of the compression; $O R_3$ and $O R'_3$ those at the beginning of the exhaust; and $O R_4$ and $O R'_4$ those at the beginning of the admission of the steam.

At first sight the present diagram might easily be taken to be more useful and correct than the diagrams which we have previously given, because Müller's diagram gives all measurements mathematically exact; but on a closer examination it shows disadvantages which are difficult to remove, so that it can be recommended only for investigations of valve-gears with *very short* eccentric-rods; for in order to obtain the measurements correctly, the figure must be drawn full-size. But as valve-gears in most cases have eccentric-rods which are very long in proportion to the eccentricity, the figure will not only occupy an inconveniently large space, but what speaks chiefly against the application of the diagram, the different circles will cut each other at very acute angles, so that the points of intersection which give the positions of the crank for the beginning of the expansion, the compression, &c., cannot be determined with the necessary exactness. The longer the eccentric-rods, i. e. the better the valve-gear, the more uncertain will be the results obtained from the diagram. But it shows still greater disadvantages when, as must almost always be the case, the diagram is drawn to a reduced scale. Finally, for those valve-gears in which the eccentricity and the angle of advance are variable, of which several will be examined farther on, it loses all utility.

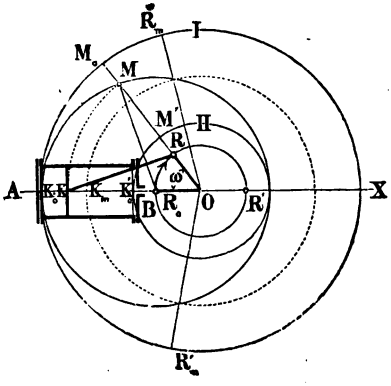
But the diagram may be applied with great advantage to an investigation of another kind, viz. to ascertain the position of the piston in the cylinder for a corresponding position of the crank.

We have always assumed in our former discussions that the length of the connecting-rod was infinite. This supposition leads in many cases to considerable errors, and the following construction, which gives the positions of the crank most correctly, is therefore to be preferred.

Let A B (Fig. 20) be the fixed steam-cylinder, O the axis of the crank-shaft, O R the crank in any position, and R K the connecting-rod of the length L, with its end K directly fastened to the piston. If the crank stands in one or the other dead point, or in the position

O R_0 or O R′, the piston will be at the end of its stroke K_0 or K'_0. If the crank O R of the radius R, has turned through the angle R_0 O R = ω, the corresponding position K of the piston is found by intersecting the line O A with an arc drawn from the point R, with the length L of the connecting-rod as a radius. K_0 K and K K'_0 are then the distances of the piston from the ends of its stroke for the corresponding position O R of the crank; and K_0 K is the distance travelled by the piston. Instead now of supposing the crank to be movable and the cylinder fixed, we take the crank as fixed in O R_0 and turn the centre line of the cylinder O A, through the angle ω, into the position O M_0. If we now, with the length of the connecting-rod L as a radius, lay off from R_0 upon the line O M_0 the point M, we shall have, as will easily be seen, M_0 M = K_0 K and M′ M = K'_0 K, if O M_0 is made = O K_0 and O M′ = O K'_0. The following rule for determining the movement of the piston is thus obtained:—Describe from the centre O of the crank-shaft with the radii O K_0 and O K'_0 the two circles I and I I. Next describe from R_0 with the length R_0 K_0 = R_0 M = L a third circle, which we may call according to Müller the distance circle. For any position O R of the crank, after producing the line O R so as to intersect the three circles, three points of intersection M_0, M, and M′ are obtained, and the distances M_0 M and M M′ give at once with mathematical precision the distances of the piston from the end points of its stroke, when the crank has travelled from the dead point through the angle ω. Bisecting the stroke K_0 K'_0 of the piston in K_m, and describing from O the fourth circle (shown in the figure in dotted lines), the lines O R_m and O R'_m, which pass through the points of intersection of this circle with the distance circle, give the two positions of the crank at which the piston is just passing through the middle of its stroke. The

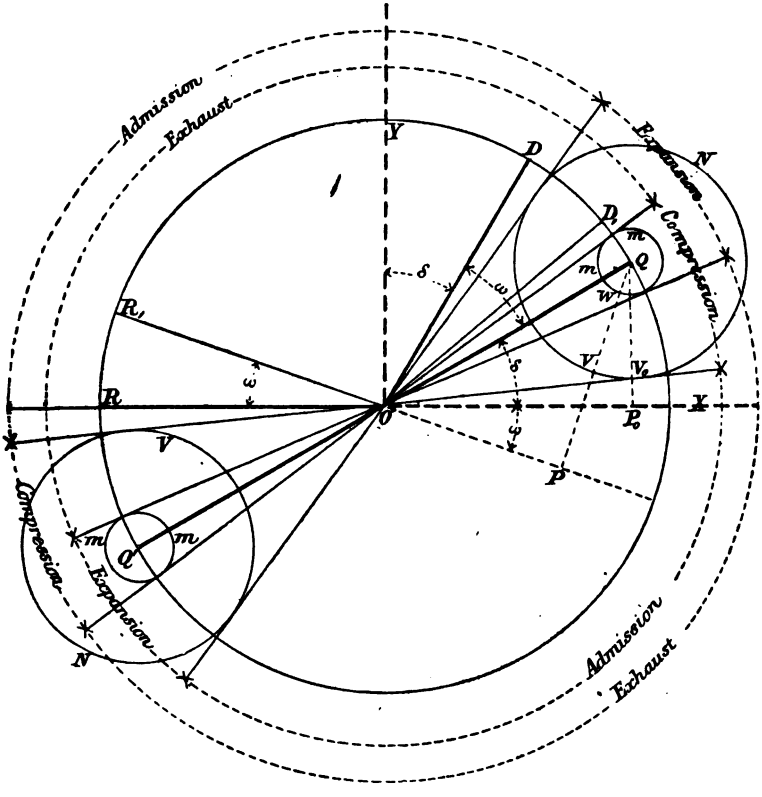
Fig. 20.



angles through which the crank has to pass from its dead points till the piston arrives at the middle of its stroke are therefore different for the backward and forward stroke; and they approach nearer to right angles the longer the connecting-rod L is in comparison to the radius R of the crank.

Bilgram's Diagram.

[This simple diagram is due to Mr. Hugo Bilgram, an American engineer. It is constructed as follows: Draw the rectangular axes O X and O Y, describe from O as a centre, with O D = r as a radius, a circle



representing orbit of eccentric, and lay off X O Q equal to angle of advance Y O D = δ; then describe from Q and Q′ as centres, and the inside and outside laps as radii, the lap circles V N and m m.

When the crank is at its first dead point it occupies the position O R

and the eccentric the position O D. Now when the crank has moved through the angle $R O R_1 = \omega$ from the dead point, the eccentricity will occupy the position $O D_1$, and, angularity of eccentric-rod being neglected, the distance of the centre of the valve from the centre of motion will be measured by the perpendicular Q P let fall from Q on the crank position $O R_1$ or its prolongation, for we have

$$Q P = O Q \sin (Q O X + X O P) = r \sin (\delta + \omega) = \xi.$$

Subtracting from this valve movement the outside lap, we get the port opening for admission of the steam, and by subtracting the inside lap we get the port opening for the exhaust. In the diagram this subtraction is performed by the lap circles, and, for the assumed crank position, these port openings are given by the distances V P and W P respectively. When the crank is at its dead point the port opening for admission or outside lead is given by the distance $V_0 P_0$.

It is evident from the diagram that the valve movement will be zero, i.e. the valve will be in its central position, when the crank occupies the position O Q or O Q′; it is also easy to see that the valve movement equals the outside lap, i.e. the admission of steam is either just beginning or just ending, when the crank is tangent to the outside lap circles; finally the valve movement is equal to the inside lap, or the exhaust is either just beginning or ending, when the crank is tangent to inside lap circles. In order to readily distinguish between the occurrences at the two steam-ports, we may employ two dotted circles as in Fig. 2, Pl. I. Here, however, both circles are placed outside the orbit of the eccentrics, the outer dotted circle representing the occurrences that take place at port nearest crank-shaft (see Fig. 1, Pl. I.), and the inner dotted circle those that take place at the port farthest from crank-shaft.—Ed.]

SECTION II.

REVERSING-MOTIONS WITH VARIABLE EXPANSION.

(*Link-motions.*)

VALVE-GEARS by which the slide-valve can be moved so that the crank-axle will run in either direction, that is, so that the engine will run forward or backward, are called "*reversing-motions.*"

These reversing-motions, which are applied in locomotive, marine, and winding engines, differ greatly in construction; by far the most important ones are those which at the same time allow of variable expansion, that is, those which allow the admission of the steam to the cylinders to be interrupted sooner or later during the stroke.

In all those valve-gears which vary the expansion by suitably moving *one* slide-valve, the motion of the valve is derived from a frame called the "*link,*" which itself is set into a rocking, and at the same time into a forward and backward motion, by one or two eccentrics. Reversing-motions with variable expansion of this kind are called "*link-motions.*"

Link-motions unquestionably belong to the most ingenious mechanical movements that occur in machinery. By simply moving a lever, some of the parts of this very simple mechanism are so shifted as to alter the movement of the *single* slide-valve, thus permitting the engine to run backward or forward with any degree of expansion, a result which it might be expected could only be obtained by very complicated mechanisms. But the link-motions now usually employed are as simple as the exact law, according to which the movement of the slide-valve takes place, is complex. The

object of this book is chiefly to demonstrate this law for the most important and best-known link-motions, and to derive from it practical and useful rules.

CHAPTER I.

STEPHENSON'S LINK-MOTION.

THIS link-motion is the one most generally used for locomotive engines. Fig. 5, Pl. II., shows the ordinary arrangement employed.

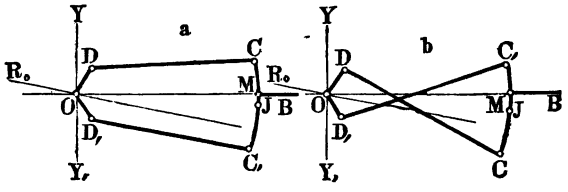
Upon the axle O are fixed the two eccentrics D and D_1, from which extend the two eccentric-rods B C and B_1 C_1, these being jointed at the ends nearest the valve to the link C C_1. In the curved slot of this link, there slides the block M, which is jointed to the guided valve-stem T. The radius of the slot in the link must be, as will be shown hereafter, equal to the length of the eccentric-rod. The link is suspended by the hanger E C_1 in such a manner, that when the engine-driver moves the reversing-lever L N, the reversing-rod G F and the lifting-arm F K E will lower or raise the hanger E C_1, and therefore also the link C C_1. The block M then slides in the slot of the link, and the driver may therefore use, according to the position of the lever L N, any point of the link for the movement of the valve-spindle T. The lever L N moves in an arc Q R, which is provided with notches,* by means of which the lever can be fixed at any position. The link is made in the two shapes represented by Figs. 6 and 7, Pl. II., and is also sometimes suspended from the upper end C or from a pin in the middle J of the link, this point being called the dead point of the link. For when the link (Fig. 5, Pl. II.) is raised so far that the point J drives the block M, and therefore the slide-valve, the distribution of the steam is such that no movement of the engine can take place; but when the

* [The notches are generally so arranged that they correspond to cut-offs which take place when the piston has travelled an integral number of inches; thus, with a stroke of 24 inches, the notches correspond to cut-offs at the piston positions 6, 9, 12, 15, 18, and 21 inches, or to the piston positions 6, 8, 10, 12, 15, 18, and 21 inches.—ED.]

driver lowers the link, the engine runs forward, because then the *forward eccentric* D principally governs the movement of the slide-valve. On the other hand, when the link is raised so far that a point below the dead point drives the block M, the engine runs backwards, the *back eccentric* then chiefly influencing the movement of the slide-valve. Now it is easy to see that when the axle turns, the link will not only receive a rocking but also a reciprocating motion, and thus transfer a peculiar movement to the block M and to the slide-valve. The latter movement is, however, so extraordinarily complicated, that it cannot be followed exactly, but only approximately, by mathematical calculation. But the results of the calculation and of the diagram agree very satisfactorily with practice.

In examining these link-motions, we have to distinguish between different cases: the eccentric-rods are either *open* or *crossed*, as represented by Fig. 21 *a* and *b*, and in either of these cases the angular advance of the two eccentrics may be *equal* or *unequal*.

FIG. 21.



Let O be the centre of the axle, O D the centre line of the forward, and O D_1 that of the back eccentric; let any point M of the link drive the valve-stem M B, and let the line of motion of the latter pass through O. If now the crank is on one of its dead points, for instance lies in the direction O R_0, and if the corresponding positions of the eccentric arms are equally inclined towards the vertical line O Y, we have *equal* angles of advance, these being $D\,O\,Y = D_1\,O\,Y_1 = \delta$; this is true, whatever the angle included between the crank and eccentric arms. If the inclinations of the eccentric arms to the vertical O Y are not equal at this position of the crank, the angles of advance will be *unequal*, and the distribution of the steam is then a different one.

In order to examine whether a link-motion works with crossed or open rods, the axle is turned so far *that the centre lines of the two eccentrics are between the vertical line* $Y\,Y_1$ *and the link.* If the eccentric-rods then occupy the positions shown in Fig. 21 *a*, they are said to be open rods, and crossed rods if they are as shown in Fig. 21 *b*. The distribution of the steam is again different in the two cases.

For further investigation we will make use of the following notation:—The amount of the eccentricities $O\,D = O\,D_1 = r$; the angular advances (both taken as equal) $D\,O\,Y = D_1\,O\,Y_1 = \delta$; the length of the eccentric-rods $D\,C = D_1\,C_1 = l$. (If the link is as in Fig. 6, Pl. II., the length is to be measured to K.) The half-length of the link, measured from the dead point J to the connecting points of the rods C and $C_1 = c$. (In the link Fig. 6, Pl. II., K J is to be put equal $K_1\,J = c$.) The distance of the slide-block M from the middle of link J, which is variable according to the position of the link, and which may be positive or negative, $= u$. Finally, let the length M B of the valve-spindles measured from the centre B of the slide-valve $= l_1$.

Theory of Stephenson's Link-motion.

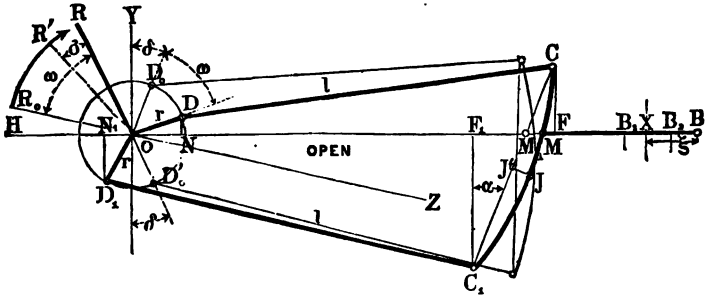
(*a*). *Determination of the Movement of the Slide-valve.*

The chief object of the theoretical investigation is to find the relation between the angular movement ω of the axle or of the crank, measured from the dead point, and the movement of the valve ξ, i. e. its distance from its centre of motion at any position of the link, or any value of u. We shall first carry out the investigation for a link-motion with open rods and equal angles of advance, and from the results obtained it is easy to determine the relations for crossed rods and unequal angles of advance.

We must first calculate, for certain corresponding values of ω and u, the inclination a of the chord of the link towards a line perpendicular to the centre line O X (Fig. 22). For this purpose we put approximately the length of the chord of the link $C\,C_1$ equal to the length of the arc, therefore $= 2\,c$, which is allowable, as the radius

for the curve of the link is always very large. In the same way we put the distance of the point M_1 in the chord from the centre of the chord J′ equal to u, i. e. = J M. If the crank is on the dead point, for instance occupies the position O R_0 (O Z representing the direction of the centre line of the steam cylinder), the different parts of the mechanism take the positions shown in Fig. 22 by the fine lines.

FIG. 22.



If the crank is now turned through an angle ω, the eccentric, the eccentric-rods, and the link will take the positions shown by the thick lines. The connecting point M of the valve-stem will not leave the line O B if the hanger is taken sufficiently long, because the latter (for a given position of the reversing-lever) during the turning of the axle, will keep the link at a constant height, at least approximately.

Let us now drop upon O B from the points C and C_1, the perpendiculars C F and C_1 F_1, and also upon O B from the points D and D_1, the perpendiculars D N and D_1 N_1, and first determine the angle $F C C_1 = F_1 C_1 C = a$.

Then

$$\sin a = \frac{F F_1}{C C_1} = \frac{O F - O F_1}{2 c}.$$

We now find the value

$$O F = O N + N F = O N + \sqrt{D C^2 - (C F - D N)^2}$$
$$= r \sin (\delta + \omega) + \sqrt{l^2 - [(c - u) \cos a - r \cos (\delta + \omega)]^2},$$

or approximately, if for the small angle a we assume $\cos a = 1$, and develop the quantity under the radical into a series (by the binomial

theorem), neglecting in the denominators all powers of l higher than the first, we have

$$\mathrm{O\,F} = r \sin(\delta + \omega) + l - \frac{c^2}{2\,l} + \frac{c\,u}{l} - \frac{u^2}{2\,l}$$
$$+ \frac{(c - u)\, r \cos(\delta + \omega)}{l} - \frac{r^2 \cos^2(\delta + \omega)}{2\,l}, \qquad (15)$$

$\sim (c-u)\cos\delta\cos\omega - \sin\delta\sin\omega)$

and next, in the same manner,

$$\mathrm{O\,F_1} = \mathrm{N_1\,F_1} - \mathrm{O\,N_1} = \sqrt{\mathrm{D_1\,C_1}^2 - (\mathrm{C_1\,F_1} - \mathrm{D_1\,N_1})^2} - \mathrm{O\,N_1},$$

or, substituting our notation,

$$\mathrm{O\,F_1} = r \sin(\delta - \omega) + \sqrt{l^2 - [(c + u)\cos a - r \cos(\delta - \omega)]^2},$$

which gives approximately

$= r\sin\delta\cos\omega - (c+u)\sin(\omega)$

$$\mathrm{O\,F_1} = r \sin(\delta - \omega) + l - \frac{c^2}{2\,l} - \frac{c\,u}{l} - \frac{u^2}{2\,l}$$
$$+ \frac{(c + u)\, r \cos(\delta - \omega)}{l} - \frac{r^2 \cos^2(\delta - \omega)}{2\,l}.$$

$\sim (c+u)\cos\delta\cos\omega + \sin\delta\sin\omega)$

If the two values of O F and O $\mathrm{F_1}$ are substituted in the equation $\sin a = \dfrac{\mathrm{O\,F} - \mathrm{O\,F_1}}{2\,c}$, we get, after sufficient reduction,

$$\sin a = \frac{r}{c} \cos\delta \sin\omega - \frac{r}{l} \sin\delta \sin\omega - \frac{u\,r}{c\,l} \cos\delta \cos\omega$$
$$+ \frac{u}{l} + \frac{r^2}{4\,c\,l} [\cos^2(\delta - \omega) - \cos^2(\delta + \omega)]. \qquad (16)$$

By this formula the inclination a of the link to the vertical line $\mathrm{C_1\,F_1}$ may be calculated for any value of ω and u.

Later on we shall be obliged to calculate this inclination a under the supposition that the dead point of the link falls on the line O X; then therefore $u = 0$, and for this case equation (16) gives at once

$$\sin a = \frac{r}{c} \cos\delta \sin\omega - \frac{r}{l} \sin\delta \sin\omega$$
$$+ \frac{r^2}{4\,c\,l} [\cos^2(\delta - \omega) - \cos^2(\delta + \omega)]. \qquad (17)$$

After these preparations it will be easy to calculate the distance of the valve centre B from the centre O of the axle. According to the figure

$$O B = O M_1 + M_1 M + M B = O F - M_1 F + M M + M B.$$

The value of O F is known according to equation (15); next $M_1 F = (c - u) \sin a$, $M B = l_1$, and $M M_1 = \frac{c^2}{2\rho} - \frac{u^2}{2\rho}$,* if we suppose that the link is curved to any radius ρ. We get thus

$$O B = O F - (c - u) \sin a + \frac{c^2}{2\rho} - \frac{u^2}{2\rho} + l_1.$$

Substituting in this equation the value of O F and that for $\sin a$ given by equation (16), we get, after sufficient reduction,

$$O B = r\left(\sin \delta + \frac{c^2 - u^2}{c\,l} \cos \delta\right) \cos \omega + \frac{u\,r}{c} \cos \delta \sin \omega$$

$$+ l + l_1 + (c^2 - u^2) \frac{l - \rho}{2\,l\,\rho}$$

$$- \frac{r^2}{4\,c\,l} [(c + u) \cos^2 (\delta + \omega) + (c - u) \cos^2 (\delta - \omega)]. \quad (18)$$

If the link-motion is to be a correct one, the valve must, for any value of u (i. e. at all positions of the reversing-lever), swing symmetrically to each side of a certain point X, the position of which is next to be ascertained.

We again start with the supposition that the valve is to be adjusted so as to give equal lead, in the manner explained on pp. 13 and 49 for the simple valve-gear.

We will first suppose the crank to be at one of its dead points,

* [If we suppose the circle, of which the link-arc is a part, to be completed, and suppose $M M_1$ to be prolonged till it intersects this circle, we shall have two chords intersecting each other at M; now, according to a well-known proposition in geometry, the products of the segments of the chords are equal, hence we have, approximately,

$$M M_1 \times 2\rho = (c - u)(c + u). \qquad \text{—Ed.}]$$

say $\omega = 0$; then the distance of the valve centre of the axle will be, according to equation (18),

$$O B_2 = r\left(\sin\delta + \frac{c^2 - u^2}{c\,l}\cos\delta\right) + l + l_1 + (c^2 - u^2)\frac{l - \rho}{2\,l\,\rho} - \frac{r^2}{2\,l}\cos^2\delta.$$

If the crank stands at the second dead point, then $\omega = 180°$, and the same equation (18) gives

$$O B_3 = -r\left(\sin\delta + \frac{c^2 - u^2}{c\,l}\cos\delta\right) + l + l_1 + (c^2 - u^2)\frac{l - \rho}{2\,l\,\rho} - \frac{r^2}{2\,l}\cos^2\delta.$$

But the centre of oscillation X must be midway between B_2 and B_3, and its distance from the centre of the axle is therefore

$$O X = \frac{O B_2 + O B_3}{2},$$

or substituting the values above given,

$$O X = l + l_1 - \frac{r^2}{2\,l}\cos^2\delta + (c^2 - u^2)\frac{l - \rho}{2\,l\,\rho}. \qquad (19)$$

In this equation u still appears, i. e. the centre of motion X is not fixed, but alters its position with the movement of the link; and this is not admissible. If the link-motion, therefore, is to fulfil all conditions, the last quantity must be *nil*, which is the case if

$$\rho = l.$$

Hence follows the important rule:—*In Stephenson's valve-gear the link must always be curved to an arc whose radius ρ is equal to the length l of the eccentric-rods.*

Under this supposition, the distance of the centre of motion from the centre of the axle is, according to equation (19),

$$O X = l + l_1 - \frac{r^2}{2\,l}\cos^2\delta, \qquad (20)$$

i. e. exactly as in the simple valve-motion (p. 13).

Further, the distance of the valve centre B from the centre of the axle for the angle ω is, according to equation (18),

$$\mathrm{OB} = r\left(\sin\delta + \frac{c^2 - u^2}{c\,l}\cos\delta\right)\cos\omega + \frac{u\,r}{c}\cos\delta\sin\omega + l + l_1$$
$$- \frac{r^2}{4\,c\,l}\left[(c+u)\cos^2(\delta+\omega) + (c-u)\cos^2(\delta-\omega)\right]; \qquad (21)$$

and finally the movement ξ of the valve from its central position

$$\xi = \mathrm{OB} - \mathrm{OX},$$

or, substituting the above values and reducing,

$$(\mathrm{II}^a) \qquad \xi = r\left(\sin\delta + \frac{c^2 - u^2}{c\,l}\cos\delta\right)\cos\omega + \frac{u\,r}{c}\cos\delta\sin\omega$$
$$+ \frac{r^2}{2\,l}\left(\cos 2\,\delta\sin\omega + \frac{u}{c}\sin 2\,\delta\cos\omega\right)\sin\omega.$$

Or calling

$$r\left(\sin\delta + \frac{c^2 - u^2}{c\,l}\cos\delta\right) = \mathrm{A}, \qquad (22)$$

$$\frac{u\,r}{c}\cos\delta = \mathrm{B}, \qquad (23)$$

$$\frac{r^2}{2\,l}\left(\cos 2\,\delta\sin\omega + \frac{u}{c}\sin 2\,\delta\cos\omega\right)\sin\omega = \mathrm{F}, \qquad (24)$$

we get

$$(\mathrm{II}^b) \qquad \xi = \mathrm{A}\cos\omega + \mathrm{B}\sin\omega + \mathrm{F},$$

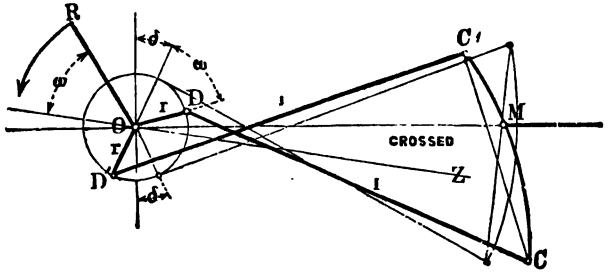
an equation of exactly the same form as that found for the simple valve-motion, only that the factors A, B, and F represent different values. The value of F given by equation (24) is again the "missing quantity," and must be exceedingly small if the centre of motion is not to be variable during the turning of the crank. As in well-constructed link-motions l is always large in proportion to r, the quantity has no great influence, and we may therefore put

$$(\mathrm{II}^c) \qquad \xi = r\left(\sin\delta + \frac{c^2 - u^2}{c\,l}\cos\delta\right)\cos\omega + \frac{u\,r}{c}\cos\delta\sin\omega.$$

From the formulas given above, we can at once obtain, without

further calculations, the formulas for the movement of the valve in the case of *Stephenson's* link-motion with *crossed eccentric-rods* (Fig. 23), as, for reasons easily to be understood,* we have only to take, in all

FIG. 23.



the formulas, the value of c as negative. We thus get for the movement of the valve with crossed eccentric-rods

$$\text{(II}^{d}\text{)} \quad \xi = r\left(\sin\delta - \frac{c^2 - u^2}{c\,l}\cos\delta\right)\cos\omega - \frac{u\,r}{c}\cos\delta\sin\omega$$
$$+ \frac{r^2}{2\,l}\left(\cos 2\,\delta\sin\omega - \frac{u}{c}\sin 2\,\delta\cos\omega\right)\sin\omega.$$

Neglecting the "missing quantity" and combining the equations of the two kinds of link-motions, we get

$$\text{(II)} \quad \xi = r\left(\sin\delta \pm \frac{c^2 - u^2}{c\,l}\cos\delta\right)\cos\omega \pm \frac{u\,r}{c}\cos\delta\sin\omega,$$

the upper sign referring to open, and the lower one to crossed rods.

(*b*). *The Curve of Centres.*

The equation obtained for the movement of the valve has again the form

$$\xi = \mathrm{A}\cos\omega \pm \mathrm{B}\sin\omega,$$

and the movements of the valve are therefore given, as shown on

* [We can pass from the case of open rods (Fig. 22), to that of crossed rods (Fig. 23), by imagining the joints C and C_1 (Fig. 22) to approach each other till they meet at the centre J of the link, and then to continue their motion past this centre till they assume the positions shown in Fig. 23. In doing this the half-length c of the link gradually changes from a plus value to zero, and then to a minus value, which last corresponds to the case of crossed rods.—ED.]

pp. 16 and 23, by chords of a circle, whose centre has the co-ordinates (Fig. 2, Pl. I.):

$$\mathrm{O\,B} = a = \frac{\mathrm{A}}{2} = \frac{r}{2}\left(\sin\delta \pm \frac{c^2 - u^2}{c\,l}\cos\delta\right),$$

and

$$\mathrm{B\,C} = b = \frac{\mathrm{B}}{2} = \frac{r\,u}{2\,c}\cos\delta.$$

But the above values of a and b are dependent upon u, i. e. upon the position of the link, hence there is a special valve circle corresponding to each position of the link. The centres of all these circles will be in a certain curve, and this curve we shall call the curve of centres. In Figs. 8 and 10, Pl. II., the different centres are marked C_0, C_1, C_2, &c., &c., the first figure being for open, and the second for crossed rods; and it will be seen that in the first case the concave side, and in the second case the convex side of the curve is turned towards the point O.

The co-ordinates for any point C_3 of the curves of centres, estimated from O as an origin, are (Fig. 8, Pl. II.), for open rods,

$$\mathrm{O\,B}_3 = \frac{r}{2}\left(\sin\delta + \frac{c^2 - u^2}{c\,l}\cos\delta\right),$$

$$\mathrm{B}_3\,\mathrm{C}_3 = \frac{u\,r}{2\,c}\cos\delta.$$

Representing the abscissa estimated from the point C_0 by x, we have

$$x = \mathrm{O\,C}_0 - \mathrm{O\,B}_3,$$

and since $u = 0$ for C_0, we get

$$\mathrm{O\,C}_0 = \frac{r}{2}\left(\sin\delta + \frac{c}{l}\cos\delta\right),$$

and therefore

$$x = \frac{r\,u^2}{2\,c\,l}\cos\delta.$$

Representing the ordinate $\mathrm{B}_3\,\mathrm{C}_3$ by y, we find from $y = \frac{u\,r}{2\,c}\cos\delta$ the value for u, and substitute it in the equation for x; thus we obtain

$$y^2 = \frac{l\,r\cos\delta}{2\,c}\,.\,x. \qquad (25)$$

The curve of centres is, therefore, a parabola, whose parameter is $\frac{l r \cos \delta}{2 c}$, and the distance between its vertex C_0 and the centre of the axle O is

$$O\,C_0 = \frac{r}{2}\left(\sin \delta + \frac{c}{l} \cos \delta\right).$$

The same equation is obtained in a similar manner for crossed rods, but the distance between the vertex and the centre O is

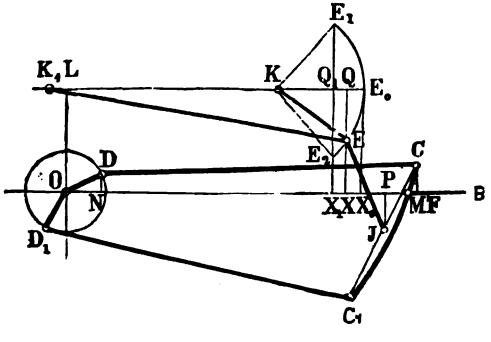
$$\frac{r}{2}\left(\sin \delta - \frac{c}{l} \cos \delta\right);$$

and the convex side of the curve is turned towards O (Fig. 10, Pl. II.).

(c). *The Suspension of the Link.*

In establishing the formula for the movement of the valve, we suppose the point M of the link, from which the valve-stem receives its motion, to move backwards and forwards exactly in a straight line O B (Fig. 24). In reality this is not the case; the link always moves a little up and down during the turning of the axle, because one of its points is fastened to a hanger, which moves round a fixed point E. The point of suspension J, therefore, moves backwards and forwards in an arc, and as the sliding-block consequently moves a little back and forth in the link, this irregularity will finally be transferred to the valve. As these irregularities may be made very great by an incorrect mode of suspension, it is one of the most important subjects of the theoretical investigation to determine the manner in which the suspension should be carried out, in order that its influence upon the movement of the valve may be as small as possible.

Fig. 24.



Two different systems of suspending the link are generally found in practice; the hanger is either fastened to the lower end C_1 of the link, or to its centre. The upper point E of the hanger always moves in an arc, whose centre K is fixed. The problem is now to ascertain the different positions of the end E of the hanger for different positions of the link, under the supposition that the link moves up and down as little as possible; this condition will be fulfilled if, under all circumstances, the point of connection of the link with the hanger moves backwards and forwards in an arc whose chord is parallel to the centre line O B of the stroke.

We shall first suppose the link to be suspended by its centre.

Let us assume the point of suspension J at the centre of the chord $C C_1$, which corresponds best with the mode of suspension practically followed, and endeavour to ascertain the horizontal motion of this point J, under the supposition that any point M of the link moves backwards and forwards in the direction of the stroke O B, and at a distance u from the dead point. Let O be the point at which the co-ordinates commence, we have then to find the abscissa O P of the point J.

It is

$$O P = O F - F P, \tag{26}$$

but according to equation (15) we have

$$O F = r \sin(\delta + \omega) + l - \frac{c^2}{2 l} + \frac{c u}{l} - \frac{u^2}{2 l} + \frac{r(c - u) \cos(\delta + \omega)}{l}, \tag{27}$$

if we neglect the small quantity

$$\frac{r^2 \cos^2(\delta + \omega)}{2 l};$$

and next we have, according to Fig. 24,

$$F P = c \sin a,$$

where a is the angle which the link chord makes with a vertical line. But the value of $\sin a$ is given by equation (16), and if we neglect the term which contains $\frac{r^2}{4 c l}$ as a factor, we get

$$\sin a = \frac{r}{c} \cos \delta \sin \omega - \frac{r}{l} \sin \delta \sin \omega - \frac{u r}{c l} \cos \delta \cos \omega + \frac{u}{l}; \tag{28}$$

therefore $r\sin(\delta+\omega) + l - \frac{c^2}{2l} + \frac{cu}{l} - \frac{u^2}{2l} + \frac{r(c-u)}{l}\cos(\delta+\omega)$

$$\text{F P} = c \sin a = r\left(\cos\delta - \frac{c}{l}\sin\delta\right)\sin\omega - \frac{u\,r}{l}\cos\delta\cos\omega + \frac{c\,u}{l}.$$

Applying to equation (26) this value, and the one obtained for O F from equation (27), we get, after a few reductions,

$$\text{O P} = l - \frac{c^2+u^2}{2\,l} + r\left(\sin\delta + \frac{c}{l}\cos\delta\right)\cos\omega + \frac{r\,u}{l}\sin\delta\sin\omega. \quad (29)$$

The abscissa O P of the point of suspension J may be calculated by this equation for any position of the link, and thus for any value of u and for any angular movement ω.

For $\omega = 0$, follows the abscissa

$$l - \frac{c^2+u^2}{2\,l} + r\left(\sin\delta + \frac{c}{l}\cos\delta\right).$$

For $\omega = 180^\circ$, however, the abscissa is

$$l - \frac{c^2+u^2}{2\,l} - r\left(\sin\delta + \frac{c}{l}\cos\delta\right).$$

The arithmetical mean of the two values gives the abscissa O X $= x$, which corresponds to the central position of the point J, and is equal to

$$x = l - \frac{c^2+u^2}{2\,l}. \quad (30)$$

But this simple equation at the same time gives the *abscissa of the upper point of suspension E for the corresponding position of the link.*

Representing the length of the hanger J E by l_2; the ordinate X E $= y$ of the suspending point E is given by

$$\text{X E} = \text{J E} - \text{J P},$$

or

$$y = l_2 - u\cos a.$$

But the angle a is so small with good proportions that we may at once assume $\cos a = 1$, and thus, approximately,

$$y = l_2 - u. \quad (31)$$

The two equations (30) and (31) give for any value of u the correct position of the point of suspension E, or the curve in which

this point has to be moved if the hurtful oscillations of the link for *all* its positions are to be reduced to a minimum.

If the link is raised so much that its dead point gives the movement to the valve, then $u = 0$, and we get for the co-ordinates of the point of suspension E_0 for this link position (Fig. 24)

$$O X_0 = x_0 = l - \frac{c^2}{2l} \quad \text{and} \quad X_0 E_0 = y_0 = l_2. \tag{32}$$

But now l is the length of the eccentric-rod and $\frac{c^2}{2l}$ is the height of the arc of the link; the difference gives at once the abscissa of the point of suspension E_0 corresponding to the dead point, and the length l_2 of the hanger is simply its ordinate; the point E_0 is thus readily found by construction.

Drawing also through E_0 the line E_0 K parallel to the centre line O B of the motion, taking E_0 as the point at which the co-ordinates for the curve E E_0 E_1 commence, and putting the abscissa $E_0 Q = v$ and the ordinate $Q E = z$, we have $E_0 Q = O X_0 - O X$, or using equations (30) and (32),

$$v = \frac{u^2}{2l}.$$

And since

$$Q E = z = Q X - E X = l_2 - y,$$

or

$$z = u,$$

there follows the required equation of the curve E E_0 E_1

$$z^2 = 2 l v, \tag{33}$$

i.e. the curve in which the point of suspension E has to be moved, if the influence of the vertical oscillations of the link upon the movement of the valve is to be reduced to a minimum, is a parabola whose parameter $= 2l$, or equal double the length of the eccentric-rod. The vertex of the parabola is at the point of suspension E_0, which corresponds to the dead point of the link, and the axis runs parallel to the centre line O B of the stroke.

The positions of the two points of suspension E_1 and E_2, corresponding to the highest and lowest positions of the link, are found by

substituting in equation (30) the greatest values of u, which we will represent by $+ c_1$ and $- c_1$, for instance in Fig. 7, Pl. II., J K $= c_1$. (In the link Fig. 6, Pl. II., u may even become equal to c.)

We thus get the abscissa belonging to both points

$$O X_1 = l - \frac{c^2 + c_1^2}{2 l},$$

or if we estimate the abscissa from E_0 upon the axis of the parabola

$$E_0 Q_1 = O X_0 - O X_1 = \frac{c_1^2}{2 l}.$$

The ordinates $Q_1 E_1$ and $Q_1 E_2$ are c_1 and $- c_1$.

As $\frac{c_1^2}{2 l}$ is always a small quantity, and the parameter $2 l$ of the parabola $E_1 E_0 E_2$ is always very great, the arc of the parabola may be replaced by an arc of a circle of the radius l, for the radius of the vertex of a parabola is equal to the half parameter.

We obtain thus the peculiar result: that the point of suspension E *is to be moved in an arc of a circle whose radius is equal to the length of the eccentric-rod*, and whose centre is at the same height above the centre line of stroke O B as the point E_0.

In order, therefore, to obtain the *true* position K_1 for the centre of the arm K E, we draw (Fig. 24) above O, at the distance O L $= E J = l_2$, the line $K_1 E_0$ parallel to O B, make $K_1 L = \frac{c^2}{2 l}$, and describe with the radius $K_1 E = l$ the arc $E_2 E_0 E_1$, *this* arc will then be the true curve, in which the point of suspension E of the hanger E J ought to move.

Before we compare the above with the constructions that obtain in practice, we will examine another common method of suspending the link, namely that in which the lower end C_1 (Fig. 25) of the link is connected with the hanger.

We start here with the same suppositions as in the former case, examining here also the horizontal movement of the point C_1 for any position of the link, and thus setting up a general formula for its abscissa O P.

According to Fig. 25 this is

$$O\,P = O\,F - F\,P.$$

O F is known from equation (27), and from the figure we see that

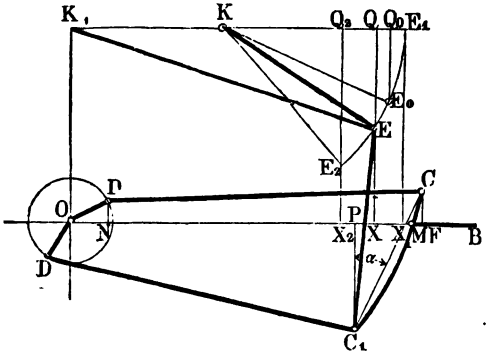
$$F\,P = 2\,c \sin a,$$

or using the value of *sin a* obtained from equation (28),

$$O\,F = r(\sin\omega + \delta) + l - \frac{c^2}{2l} + \frac{cu}{l} - \frac{u^2}{2l} + r(c - u)\cos(\omega + \delta)$$

$$F\,P = 2\,r\left(\cos\delta - \frac{c}{l}\sin\delta\right)\sin\omega - 2\,\frac{u}{l}\cos\delta\cos\omega + 2\,\frac{c\,u}{l}.$$

FIG. 25.



Hence follows, after substitution and reduction,

$$O\,P = l - \frac{(c+u)^2}{2\,l} + r\left[\sin(\delta - \omega) + \frac{c+u}{l}\cos(\delta - \omega)\right]. \quad (34)$$

The abscissa O P of the lower end C_1 of the link may therefore be calculated from this equation for any position of the link, i. e. for any value of u and for any angular movement ω.

For $\omega = 0$, the value of the abscissa is

$$l - \frac{(c+u)^2}{2\,l} + r\left[\sin\delta + \frac{c+u}{l}\cos\delta\right].$$

For $\omega = 180°$ the abscissa is

$$l - \frac{(c+u)^2}{2\,l} - r\left[\sin\delta + \frac{c+u}{l}\cos\delta\right].$$

The arithmetical mean of these two quantities gives the abscissa $O X = x$, which corresponds to the *central position* of the point C_1; this mean is

$$O X = x = l - \frac{(c + u)^2}{2 l}. \tag{35}$$

But this is at the same time the *abscissa of the upper point of suspension* E, for the hanger $E C_1$ has to hang vertically, when C_1 passes through its centre of motion. The corresponding ordinate y is then

$$X E = y = E C_1 - C_1 P,$$

or

$$y = l_2 - (c + u) \cos a.$$

But as a is nearly always very small, we may again put approximately $\cos a = 1$, and then get

$$y = l_2 - (c + u). \tag{36}$$

The combination of the two equations (35) and (36), by eliminating $(c + u)$, gives at once the equation of the curve $E E_0 E_1$ through which the point of suspension is to be moved when this method of suspending the link is adopted. We now have

$$(l_2 - y)^2 = 2 l (l - x), \tag{37}$$

and this is again *the equation of a parabola* whose parameter is $2 l$, but whose vertex measured from O has $O X_1 = l$ as abscissa and l_2 as ordinate. It is therefore very easily found by construction.

If the link can be raised so high that the point C_1 coincides with the line of the stroke, then $u = - c$, and the equations (35) and (36) then give the co-ordinates of the point of suspension E_1:

$$O X_1 = l$$

and

$$X_1 E_1 = l_2,$$

i. e. the co-ordinates of the vertex of the parabola. The *highest* position E_1 of the point of suspension corresponds therefore, in this second method of suspending the link, with the vertex of the parabola, and the curve in which the suspending point E has to

move forms only *one* branch of the parabola, while in the first method of suspension the point E has to be moved through both branches.

Again, estimating the co-ordinates of the point E from the vertex E_1, and representing the abscissa E_1 Q by v, the ordinate Q E by z, the equation (37) may be written as follows:

$$z^2 = 2\,l\,v, \tag{38}$$

for v is equal to $l - x$, and z equal to $l_2 - y$.

Taking also here c_1 and $-c_1$ as the greatest values of u, we get the co-ordinates of the point of suspension measured from E_1:—

For the *highest* position of the link:

$$v_2 = \frac{(c - c_1)^2}{2\,l}; \qquad z_2 = c - c_1.$$

For the dead point:

$$v_0 = \frac{c^2}{2\,l}; \qquad z_0 = c.$$

For the *lowest* position of the link:

$$v_1 = \frac{(c + c_1)^2}{2\,l}; \qquad z_1 = c + c_1.$$

The total horizontal motion of the point of suspension E is thus equal to $v_1 - v_2 = \frac{2\,c\,c_1}{l}$, when the link is moved from the highest to the lowest position.

We may also here substitute for the arc of the parabola E_1 E an arc of a circle *whose radius is equal to the half parameter, or equal to the length of the eccentric-rod,* but whose centre K_1 lies at the same height above the centre line of the motion as that position of the point of suspension E_1 which corresponds with the highest, and not with the central position of the link. The true centre K_1 lies in this case exactly above the centre O of the axle (Fig. 25).

Comparing the result of this theoretical investigation, viz. that the upper point of suspension of the hanger should move in a circular arc whose radius is equal to the length of the eccentric-rod, with the proportions adopted in practice, we find that this radius is generally much smaller (K E in Fig. 5, Pl. II.). This explains

why, in some locomotive link-motions, the link *in certain positions* moves up and down in such a manner that for these positions there can be no regular movement of the valve and distribution of the steam. The above rule could be applied in many cases by moving the point of suspension E either in a corresponding guide, or by really suspending the link from the end of a lever of the length l, this lever being raised or lowered by the bell-crank lever E K F; but in most cases circumstances prevent the radius of the arc from being made as long as the eccentric-rod. Therefore, in designing the valve-gear, the points E_1, E_0, and E_2 must be shifted parallel to the direction O B of the stroke, till the radius of the circle which passes through these points has the practicable length; but it is always desirable to previously fix the right position of these points, and to make the lifting-arm K E of the bell-crank lever E K F (Fig. 5, Pl. II.) and the hanger as long as circumstances will possibly allow.

In some engines we find the bell-crank lever below the link; to this case the above investigations are also applicable.

Practical Application of the Diagram.

In the description of *Stephenson's* link-motion, upon p. 68, the manner in which the different dimensions were to be understood and designated was duly stated, and only with regard to the value of u is it now necessary to make a few remarks; u expresses the distance between the dead point J and that point M of the link (Fig. 5, Pl. II.) which gives motion to the valve. Of course this value differs according to the position of the hand-lever: it is positive in the following formula when M lies above J, but negative when the link is raised so far that the point M lies below J. If the link is raised or lowered to the utmost, u has the greatest value, which we call c_1; this is the case in the link shown in Fig. 7, Pl. II., where $J K = J K_1 = c_1$, c_1 being less than c; but in the link Fig. 6, Pl. II., $J K = c_1 = c$. The length J K or $J K_1$ may now be divided into a number of equal parts, which are numbered above and below the dead point J, and we may say the slide-block

is in the 1st, 2nd, 3rd . . . grade fore-gear or back-gear. In practice, these grades are not marked upon the link, but upon the arc Q R (Fig. 5, Pl. II.), where each notch corresponds with a certain grade.* If c_1 is divided into n parts, and if the link is in the mth grade, then

$$u = \frac{m}{n} \cdot c_1.$$

The theoretical investigation gave for the movement of the valve ξ in *Stephenson's* link-motion

$$\xi = r\left(\sin\delta \pm \frac{c^2 - u^2}{c\,l}\cos\delta\right)\cos\omega \pm \frac{r\,u}{c}\cos\delta\sin\omega, \qquad (39)$$

where the upper sign refers to open and the lower one to crossed rods.

Placing

$$r\left(\sin\delta \pm \frac{c^2 - u^2}{c\,l}\cos\delta\right) = \mathrm{A},$$

$$r\,\frac{u}{c}\cos\delta = \mathrm{B},$$

we may write more simply

$$\xi = \mathrm{A}\cos\omega + \mathrm{B}\sin\omega. \qquad (40)$$

For a given position of the link, therefore, u is a known quantity; and if the other dimensions are also known, the two values of A and B can be calculated. Now laying off from O on the centre line $\mathrm{X\,X_1}$ (Fig. 2, Pl. I.) the two values $\mathrm{O\,B} = \mathrm{O\,B_1} = \frac{1}{2}\mathrm{A} = a$, drawing the perpendiculars B C and $\mathrm{B_1\,C_1} = \frac{1}{2}\mathrm{B} = b$, and describing from C and $\mathrm{C_1}$ circles with the radii C O and $\mathrm{C_1\,O_1}$, we get, as theory has shown, the valve circles; these, as in the simple valve-motion, fully explain the movements of the valve and the distribution of the steam when we describe from O, with the outside and inside lap as radii, the circles $\mathrm{V\,V_2}$ and $\mathrm{W\,W_2}$. The upper circle is for the

* See footnote, p. 66.

forward stroke, the lower one for the return stroke. As the proportions are the same for both strokes, we need examine the upper circle only.

The valve circle thus obtained gives the distribution of the steam for the assumed position of the link, i.e. for the one grade only; a special circle belongs to each grade, for which the co-ordinates $\frac{1}{2}$ A and $\frac{1}{2}$ B of the centre are calculated in the same manner. The designer has only to substitute the known dimensions and the corresponding value of u in the above equations.

If the link is raised so far that its dead point lies above the centre line of the motion, negative values of u must be substituted in the formulas, and the circles take the position shown in Fig. 4, for $\frac{1}{2}$ B becomes negative, and thereby indicates that the crank moves in the opposite direction; the engine therefore runs the other way. The distribution of the steam is in this case exactly the same as during the forward stroke, consequently it is only necessary to examine the latter. Therefore, only draw the upper valve circle, and suppose the imaginary crank to turn from O X (Fig. 2, Pl. I.) in the direction of the arrow.*

The manner in which, after the valve circles are drawn, the distribution of the steam can be ascertained from the points of intersection has been stated fully in explaining Fig. 2, Pl. I., on p. 23, and need not be repeated; we will now illustrate by examples the peculiarities of this link-motion.

We shall first examine a link-motion with *open eccentric-rods and equal angles of advance*, and at once assume particular values for the dimensions.

Problem.—Let the eccentricity of each eccentric be $r = 0 \cdot 060^{m}$ ($2 \cdot 36''$), the angles of advance $\delta = 30°$, the length of the eccentric-rods $l = 1 \cdot 400^{m}$ ($55 \cdot 1''$), and the half-length of the link $c = 0 \cdot 150^{m}$ ($5 \cdot 9''$). Let the link have four grades of expansion for the fore-gear, and four for the back-gear, and let it be capable of being raised or

* See footnote, p. 23.

lowered so far that u becomes equal to c; and let the link be of the form represented in Fig. 6, Pl. II.

Let the outside lap be $e = 0 \cdot 024^m$ ($\cdot 94''$), and the inside lap $i = 0 \cdot 007^m$ ($\cdot 27''$).

All questions, relating either to the various grades of expansion or to the dead point, are to be answered.

According to the above, the valve circles for the four grades of expansion are first to be determined; the co-ordinates of their centres for *open* eccentric-rods are to be calculated according to the formulas

$$a = \frac{1}{2} r \left(\sin \delta + \frac{c^2 - u^2}{c\, l} \cos \delta \right)$$

and

$$b = \frac{1}{2} \frac{r\, u}{c} \cos \delta.$$

For the 4th grade of expansion $u = c$, and then according to the above formulas,

$$\mathrm{O\,B_4} = \frac{1}{2} r \sin \delta$$

and

$$\mathrm{B_4\,C_4} = \frac{1}{2} r \cos \delta \text{ (Fig. 8, Pl. II.)},$$

i.e. the movement of the valve takes place exactly as if the link were not there, and as if the forward eccentric only acted upon the valve, for the co-ordinates are the same as those obtained for the simple valve-motion.

We therefore find, in Fig. 8, Pl. II., the centre $\mathrm{C_4}$ of the valve circle for the fourth grade of expansion, by laying off

$$\mathrm{O\,B_4} = \frac{1}{2} r \sin \delta = \frac{1}{2} \cdot 0 \cdot 06 \cdot \sin 30^\circ = 0 \cdot 015^m \ (0 \cdot 59'')$$

and

$$\mathrm{B_4\,C_4} = \frac{1}{2} r \cos \delta = \frac{1}{2} \cdot 0 \cdot 06 \cdot \cos 30^\circ = 0 \cdot 026^m \ (1 \cdot 02''),$$

or by making the angle $\mathrm{Y\,O\,C_4} = 30^\circ$ and $\mathrm{O\,C_4} = \frac{1}{2} r = 0 \cdot 03^m$ ($1 \cdot 18''$). The corresponding valve circle is marked IV.

For the third grade of expansion $u = \frac{3}{4} c$, and substituting this value as well as the other known values in the above equations, we get the co-ordinates of the valve circle centre C_3:

$$O B_3 = 0 \cdot 0162^m \ (0 \cdot 64'')$$

and

$$B_3 C_3 = 0 \cdot 0195^m \ (0 \cdot 77'');$$

for the second grade $u = \frac{2}{4} c$, therefore the co-ordinates of the valve circle centre C_2 are

$$O B_2 = 0 \cdot 0171^m \ (0 \cdot 67'')$$

and

$$B_2 C_2 = 0 \cdot 013^m \ (0 \cdot 51'');$$

for the first grade $u = \frac{1}{4} c$, the corresponding co-ordinates are

$$O B_1 = 0 \cdot 0176^m \ (0 \cdot 69'')$$

and

$$B_1 C_1 = 0 \cdot 0065^m \ (0 \cdot 26'');$$

finally, for the *dead* point $u = 0$; hence

$$O B_0 = \frac{1}{2} r \left(\sin \delta + \frac{c \cos \delta}{l} \right) = 0 \cdot 0178^m \ (0 \cdot 70'')$$

and

$$B_0 C_0 = 0,$$

i. e. the centre of the valve circle for the dead point lies at C_0 or on the axis O X itself. Valve circles are now drawn from the centres C_0, C_1, C_2, C_3, C_4 with the radii C_0 O, C_1 O, C_2 O, C_3 O, C_4 O, and these are designated in the figure by 0, I., II., III., IV., according to the corresponding grades of expansion. If we describe from O a circle, with the radius $O V = e = 0 \cdot 240^m$ equal to the outside lap, and another one with a radius equal to the inside lap $O W = i = 0 \cdot 007^m$, all questions may be easily answered by means of the diagram.

Let the crank be first on the dead point, that is, let the ideal crank be in the position O R, and then suppose it turned in the

direction of the arrow through any angle $R O R_1 = \omega$, then we obtain the following table (see Fig. 8, Pl. II.):—

	Dead Point.	Grade of Expansion.			
		1	2	3	4
Movement of the valve	$O P_0$	$O P_1$	$O P_2$	$O P_3$	$O P_4$
Port opening for admission	$V P_0$	$V P_1$	$V P_2$	$V P_3$	$V P_4$
Port opening for exhaust	$W P_0$	$W P_1$	$W P_2$	$W P_3$	$W P_4$

The required values are thus obtained full-size for all degrees of expansion. But the diagram also gives the following linear quantities, to which we have added the measurements taken from the diagram for the present special case:—

	Dead Point.	Grade of Expansion.			
		1	2	3	4
	m	m	m	m	m
Outside lead	$V_1 p_0$ (0·0115)	$V_1 p_1$ (0·0110)	$V_1 p_2$ (0·0100)	$V_1 p_3$ (0·0085)	$V_1 P_4$ (0·0055)
Inside lead	$W_1 p_0$ (0·0285)	$W_1 p_1$ (0·0280)	$W_1 p_2$ (0·0270)	$W_1 p_3$ (0·0255)	$W_1 P_4$ (0·0220)
Greatest valve movement.. ..	$O p_0$ (0·0355)	$O c_1$ (0·0375)	$O c_2$ (0·0430)	$O c_3$ (0·0510)	$O c_4$ (0·0595)
Greatest port opening for admission	$V_1 p_0$ (0·0115)	$b_1 c_1$ (0·0135)	$b_2 c_2$ (0·0190)	$b_3 c_3$ (0·0270)	$b_4 c_4$ (0·0355)
Greatest port opening for exhaust	$W_1 p_0$	$a_1 c_1$	$a_2 c_2$	$a_3 c_3$	$a_4 c_4$
Crank angle ω corresponding to greatest port opening	0	$X O c_1$	$X O c_2$	$X O c_3$	$X O c_4$

Practical men generally attribute special importance to the amount of the outside and inside lead, i. e. to the openings of the port for the admission and release of steam when the crank is at the dead points.

According to the above, $V_1 p_4$, $V_1 p_3$, &c., give the lead for the 4th, 3rd, &c., grade of expansion; now, either from the above Table or from Fig. 8, Pl. II., it will be seen that the lead increases with the expansion, i. e. increases the nearer that point of the link which governs the valve approaches the dead point of the link.

This peculiarity, which only appears in *Stephenson's* link-motion

with open rods, is well known in practice, and is very often regarded as a defect of *Stephenson's* link-motion.

In the present case the outside lead $V_1 p_2$ at the 2nd grade is twice as great as $V_1 p_4$, that at the 4th grade. The lead $V_1 p_0$ for the dead point is the greatest, and this point therefore gives the greatest port opening for admission at the commencement of the stroke.

The reason for the variation of the lead is only to be found in the quantity,

$$\frac{c^2 - u^2}{c\,l} \cos \delta,$$

contained in the formula for the abscissa a of the centre of the valve circle. The smaller this quantity, i. e. the shorter the link and the longer the eccentric-rod, the less variable is the lead.

The greatest port opening for admission is a matter of importance; let us suppose the width of the port to be $0 \cdot 030^m$ ($1 \cdot 18''$), then the fourth line of the above Table shows that the port is fully opened only when the link is way down, that is, at the 4th grade, and that the opening becomes smaller the more the link is raised, until finally, at the dead point, the port is at the most only opened one-third of its width.

This narrowing of the port is a disadvantage, and this is the reason why very broad valves are employed in all link-motions.

According to the diagram the port opening for the exhaust is only a little narrowed at the dead point, at all other grades of expansion it is fully opened when the valve is at the extremity of its stroke.

But all questions relating to the principal positions of the crank for each grade of expansion are also answered by the diagram if we determine the points of intersection of each valve circle with the lap circles, and proceed as with the simple valve-motion in Fig. 2, Pl. I.

The upper part of the diagram is obtained in this manner. The vertical lines represent the chief positions of the piston for all degrees of expansion. Moreover, the mode of determining the position of the piston for the 2nd grade is shown by dotted lines. If the piston travels away from crank-shaft, the admission of steam to cylinder end *nearest* crank-shaft ceases at a, and at this point expansion begins; at b the exhaust from cylinder end *farthest* from crank-shaft ceases and compression begins; at c the exhaust from cylinder end *nearest* crank-shaft begins; and at d the admission of steam to

cylinder end *farthest* from crank-shaft takes place.* It will be at once seen from the figure, that the compression and the release of the steam, as well as its admission on the other side of the piston, begin sooner, the greater the expansion, i. e. the nearer the sliding-block is to the dead point of the link.

A peculiar distribution of steam takes place at the dead point; for the compression begins before the piston has travelled half of its stroke, and shortly afterwards the steam is released from the cylinder end nearest crank-shaft, provided the engine is running. The dead point of the link, therefore, produces a distribution so unsuitable that it is not able to effect a movement of the engine. Other peculiarities of the dead point may be easily found from the diagram by examining its valve circle O.

The cut-off and compression for any degree of expansion may also be ascertained from the upper part of Fig. 8, Pl. II. Let us assume that the piston positions *a*, *b*, &c., have been determined with regard to the length of the connecting-rod in the manner previously explained, and not as is done on the Plate for want of space, by dropping upon H K perpendiculars from the chief positions of the crank-pin; then the required ratios may be very easily found. At *a* admission of steam ceases, therefore we have H *a* : H K the *cut-off* or *ratio of expansion.* At *b* the compression of steam begins, because then the steam ceases to exhaust from cylinder end *farthest* from crank-shaft, therefore H *b* : H K is the *ratio of compression.* The stroke of the piston in the diagram is H K = 1 decimetre (3·937″), and H *a* and H *b* expressed in decimetres at once give the required ratios; the diagram gives for the present special case—

	Dead Point.	Grade of Expansion.			
		1	2	3	4
Cut-off or ratio of expansion	0·165	0·335	0·535	0·690	0·800
Ratio of compression ..	0·390	0·580	0·730	0·845	0·905

* [We suppose the engine placed as in Fig. 5, Pl. II., and to have a right-handed rotation; if we view this engine from the side opposite to that shown in Fig. 5, Pl. II. the actual directions of both crank and piston will agree with those given in the diagram Fig. 8, Pl. II.—Ed.]

When the length of the connecting-rod is taken into account, these ratios will be slightly different for the forward and return strokes of the piston.

The author has made experiments on a model of the dimensions assumed in the example, and the results obtained have completely corresponded with those of the diagram.

The values which have been found for the outside lead are the only ones given here—

	Dead Point.	Number of the Grade of Expansion.			
		1	2	3	4
Forward stroke	11·5 (0·45″)	11·0 (0·43″)	9·7 (0·38″)	8·2 (0·32″)	5·5 (0·211″)
Backward stroke	11·5 (0·45″)	10·7 (0·421″)	9·5 (0·374″)	7·4 (0·29″)	5·7 (0·224″)
Medium	11·5 (0·45″)	10·8 (0·424″)	9·6 (0·377″)	7·8 (0·30″)	5·6 (0·22″)

A comparison with the figures of the upper Table shows that there is almost perfect agreement; the differences are only fractions of a millimetre.

If we assumed *crossed* eccentric-rods in the above example, the centres of the valve circles for the higher degrees of expansions will fall in the same order upon the left-hand side of the ordinate $B_4\ C_4$, as shown in Fig. 10, Pl. II. In this figure the same valve-gear dimensions were assumed as in Fig. 8, Pl. II., and the co-ordinates of the centres of the valve circles were calculated according to the formulas

$$a = \frac{1}{2} r \left(\sin \delta - \frac{c^2 - u^2}{c\,l} \cos \delta \right),$$

$$b = \frac{1}{2} \frac{r\,u}{c} \cos \delta,$$

for in the general formula (39), the lower sign is to be taken for crossed eccentric-rods.

It will be at once seen from diagram Fig. 10, Pl. II., that the fault in this arrangement is just the opposite of that previously

described, the lead, inside as well as outside, *decreasing, the higher the degree of expansion.* According to Fig. 10, we have for the

4th grade, the lead	$V_1 p_4$	$= 5 \cdot 5^{mm}$ $(0 \cdot 216'')$
3rd „ „	$V_1 p_3$	$= 3 \cdot 6^{mm}$ $(0 \cdot 1417'')$
2nd „ „	$V_1 p_2$	$= 1 \cdot 8^{mm}$ $(0 \cdot 07'')$
1st „ „	$V_1 p_1$	$= 0 \cdot 8^{mm}$ $(0 \cdot 03'')$
Dead point ..	$V_1 p_0$	$= 0 \cdot 4^{mm}$ $(0 \cdot 01'')$

i. e. the opening of the ports at the dead point takes place here only for a moment. This decrease of lead may become so great, that no outside lead at all will be given when the outside lap $O V_1$ is taken quite large. The upper part of the diagram, which gives the chief positions of the piston for the various grades of expansion, shows that the distribution of the steam in the present case is very different from that in the previous one. The most remarkable fact here is, that the position *d* of the piston is almost the same at all grades, i. e. that the pre-admission of the steam almost always takes place at the same point of the stroke.

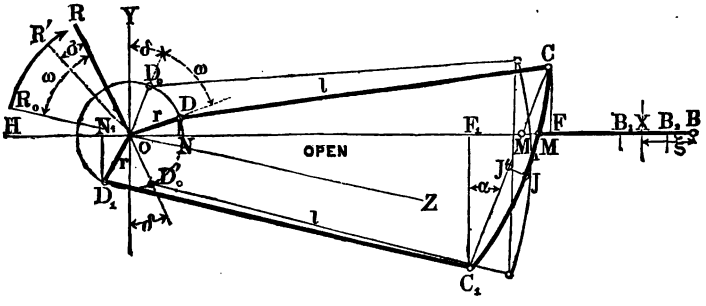
How, on the other hand, the different dimensions of a link-motion required to fulfil given conditions with regard to the distribution of the steam may be determined need not be examined here, as the mode of proceeding, according to the above diagram, does not present any further difficulties. (See remarks at the end of the first chapter.)

The variation of the lead in *Stephenson's* link-motion is generally considered to have a disadvantageous influence upon the effect of the steam, and this is why practical men have endeavoured to invent link-motions in which the lead shall be constant or nearly constant for all grades of expansion. We shall examine such valve-gears more minutely hereafter, and shall now only describe the changes which have been introduced into *Stephenson's* link-motion in order to diminish the variation of the lead as much as possible. A method which has been lately applied very frequently is remarkable for its simplicity, viz. *the eccentrics are simply fixed with different angles of advance.* This arrangement attains its aim almost completely, i. e. by this means the lead can be made nearly constant for the forward motion of the locomotive; but experience shows that this improve-

ment takes place at the expense of the backward motion, i. e. that the lead for the back-gear is made by this arrangement so much worse, so much more variable; this also our diagram shows at once. *Phillips* first carried out the calculation under the supposition that the angles of advance were variable, but he only came to the conclusion that in this manner *a better expansion might be obtained;* this is perfectly true, but it is of no great importance in practice, as equally good results can be obtained with equal angles of advance by a correct choice of dimensions. But the chief reason above stated for the application of different angles of advance is not mentioned at all by *Phillips;* the other writers, *Weisbach, Zech,* and *Redtenbacher,* moreover, speak only of the more simple case.

Before beginning these investigations, we must remember that the angle of advance is that angle which the centre lines of the eccentrics O D and O D_1 form with the perpendicular OY let fall on line O B of the stroke (Fig. 26), *when the crank* O R *occupies the dead point,* i. e. occupies the position O R_0, when O Z represents the centre line of the cylinder. At first open rods are assumed, as in Fig. 26.

FIG. 26.



In this case O D is the forward and O D_1 the back eccentric. Now, in order to get a less variable lead, the designers set *back* the crank O R by a certain angle R O R′ = σ, the crank taking some such position as O R′, shown in the figure by a dotted line; but this alters the quantities which we have called the angles of advance; for if the new position O R′ of the crank passes through the dead point,

i. e. occupies the line O R_0, then the eccentricity O D of the forward eccentric forms with the vertical line O Y the angle $\delta + \sigma$, and the eccentricity O D_1 of the back eccentric the angle $\delta - \sigma$; with this vertical the angles of advance are therefore *different.* But it will be at once seen, that such changing of the position of the crank does *not at all* alter the relative arrangement of the different parts of the valve gear, and therefore does *not at all* alter the movement of the valve; the only change is in the relative positions of piston and valve.

If we call O R the imaginary crank, O R′ the actual one, and R O R′ $= \sigma$ the angle through which the one must travel before it reaches the position of the other, then the diagram in Fig. 8, Pl. II., which has been drawn for *Stephenson's* link-motion (open rods), will also be correct for the present case, as it relates only to the movements of the valve. But the distribution of the steam will be altered and will be different for the forward and return stroke; for this reason we have drawn Fig. 9, Pl. II., which gives the diagram for the return stroke. This diagram was constructed in accordance with preceding propositions, by drawing downwards, from the axis of abscissas O R, the ordinates for the centres of the valve circles. Returning to Fig. 8, Pl. II., and, for equal angles of advance, assuming the crank at O R or on its dead point, the openings of the steam-ports at this position of the crank for the different degrees have been shown to be $V_1 p_0$, $V_1 p_1$, $V_1 p_2$, &c. Now, supposing the crank to have been turned through the angle R O R_1, the diagram gives for the different grades the openings V P_0, V P_1, V P_2, &c., for admission, belonging to the corresponding positions of the crank; in the two cases, therefore, the openings for the different grades of expansion are very different. A close examination of the diagram shows that between these two positions of the crank there is one at which these openings at the different grades differ least; this is the position of the crank which is represented by the line O R_0 and passes through q_4, at which point the valve circle for the highest grade of expansion cuts the valve circle belonging to the dead point. Making therefore such an arrangement that the crank is on its dead point when it occupies the position O R_0, the openings of the

steam-ports for all grades differ least, or in other words, *the lead is least variable, when the crank is on its dead point in the position* O R_0. The actual crank must therefore *follow behind the imaginary one as much as the angle* R O $R_0 = \sigma$. The diagram Fig. 8, Pl. II., drawn to a larger scale for the sake of greater exactness, now gives the following values:—

I. Lead with *equal* angles of advance.

Forward and return stroke.

Fig. 8, Pl. II.

4th grade of expansion	$V_1 p_4 =$	$5 \cdot 5^{mm}$ (0·216″)
3rd „ „	$V_1 p_3 =$	$8 \cdot 5^{mm}$ (0·337″)
2nd „ „	$V_1 p_2 =$	$10 \cdot 0^{mm}$ (0·393″)
1st „ „	$V_1 p_1 =$	$11 \cdot 0^{mm}$ (0·43″)
Dead point	$V_1 p_0 =$	$11 \cdot 5^{mm}$ 0·45″).

II. Lead with *unequal* angles of advance

(the actual crank following behind the imaginary crank as much as the angle R O R_0).

	Forward Stroke. Fig. 8, Pl. II.	Return Stroke. Fig. 9, Pl. II.
4th grade of expansion	$V_0 q_4 = 11 \cdot 3^{mm}$ (0·44″)	$V_0 q_4 =$ 0
3rd „ „	$V_0 q_3 = 12 \cdot 3^{mm}$ (0·483″)	$V_0 q_3 = 3 \cdot 7^{mm}$ (0.143″)
2nd „ „	$V_0 q_2 = 12 \cdot 7^{mm}$ (0·497″)	$V_0 q_2 = 7 \cdot 0^{mm}$ (0·26″)
1st „ „	$V_0 q_1 = 12 \cdot 3^{mm}$ (0·483″)	$V_0 q_1 = 9 \cdot 5^{mm}$ (0·36″)
Dead point	$V_0 q_0 = 11 \cdot 3^{mm}$ (0·44″)	$V_0 q_0 = 11 \cdot 4^{mm}$ (0·44″).

It follows from this that with equal angles of advance the lead increases from the 4th grade to the dead point, as much as 6^{mm} (0·24″); but if the crank is set back as much as the angle R O $R_0 = \sigma$, the lead remains nearly constant for the forward stroke; it is equal and also smallest for the dead point and the last grade, and largest at the 2nd grade; but the entire difference amounts to only $1 \cdot 4^{mm}$ (0·05). (It need not appear strange, that the lead above given is larger than is usual in practice, for in order to facilitate comparisons we preferred to retain the same data. Properly, the *outside lap* ought

to have been taken a little larger with the altered position of the crank).

It will be seen, therefore, that the designers reach their aim with surprising perfection, of course only for the forward motion; but the variations are so much more unfavourable for the backward motion; the lead decreases here from the dead point to the 4th grade as much as $11 \cdot 4^{mm}$ ($0 \cdot 45''$). Sometimes there will be no lead at all at the last grades, the admission of the steam beginning when the crank is *past* its dead point. But all this is only applicable to *open* eccentric-rods; it will be a little different for crossed ones as will be shown later.

The angle through which the crank in the case of open eccentric-rods must be set back, has hitherto been found by experimenting on models, the eccentric being turned until it was ascertained at which positions of the eccentrics and the crank the leads varied least; but our diagram gives the corresponding positions instantaneously and with mathematical precision. Before showing by means of an example how the aim is best reached, a few remarks may be made respecting the manner in which the distribution of the steam takes place. Taking again Fig. 8, Pl. II., and examining the distribution of the steam, say for the 2nd grade of expansion, we determine the points of intersection of the valve circle with the lap circles. Assuming equal angles of advance, we know that V_3 O R gives the angle at which the crank stands before the dead point when the pre-admission of the steam begins. But with different angles of advance, the angle V_3 O R_0 is larger than before by the angle R O $R_0 = \sigma$. The admission of the steam, therefore, takes place earlier during the forward motion *with different angles of advance than with equal angles of advance.* In the case of the backward motion it is just the reverse; in this case we have (Fig. 9, Pl. II.)

angle V_3 O R_0 less than angle V_3 O R.

The same is true of the beginning of the exhaust, for in Fig. 8:—

W_3 O R represents, with equal angles of advance, the angle at which the crank stands *before* the dead point when the exhaust

begins in the cylinder end farthest from crank-shaft. Now, during the forward motion and with the altered position of the crank, we have

$$W_3 O R_0 \text{ greater than } W_3 O R,$$

but during the backward motion

$$W_3 O R_0 \text{ is less than } W_3 O R \text{ (Fig. 9).}$$

It may therefore be asserted, that the distribution of the steam, with respect to the beginning of the admission and exhaust, which results from altering the position of the crank, *is deteriorated for the forward motion, and improved for the backward motion,* because a very early beginning of the admission and release is found to be disadvantageous. But this disadvantage may be much reduced by suitably increasing the outside lap.

O V_2 (Fig. 8) shows the position of the crank at the beginning of the expansion. Now as

$$\text{angle } V_2 O R_0 \text{ is less than angle } V_2 O R,$$

expansion will take place earlier during the forward motion when the angles of advance are different. But if the outside lap is also increased, as it ought to have been in the present case, then the expansion can begin still earlier. Therefore, other things being equal, different angles of advance give *an earlier cut-off of the steam, and expansion will be more effectual* than with equal angles of advance. It has already been stated that this is not an advantage of much importance.

But for the backward motion just the opposite takes place, the cut-off of the steam beginning later for the same grade, because

$$V_2 O R_0 \text{ is greater than } V_2 O R \text{ (Fig. 9).}$$

Finally, O W_4 is the position of the crank at the beginning of the compression in cylinder end farthest from crank-shaft.

After what has preceded it is easy to see from the diagram that the compression of the steam for the same grade of expansion begins, with

different angles of advance, *earlier* during the forward motion and *later* during the backward motion, than with equal angles. If compression were really so disadvantageous as is generally supposed, the distribution of the steam for the forward motion would in this respect be *deteriorated instead of improved* by the adoption of different angles of advance; the whole advantage would then only consist in having a lead which is constant for all grades of expansion. Nevertheless designers now very often choose different angles of advance, and engine-drivers usually run the engines with a high grade of expansion, raising or lowering the link very little for ordinary trains. All this shows that the effect of the steam, notwithstanding the high compression, cannot be so unfavourable as is usually supposed. (See article by *Reuleaux*, Civil Ing., vol. iii. p. 43.)

The manner of finding the angle through which, when open eccentric-rods are used, the crank has to be set back in order to make the lead as little variable as possible, has already been explained; but an example will show still more the advantages of the diagram.

Problem.—A link-motion for locomotives, according to *Stephenson's* system, with open eccentric-rods, is to be constructed in such a manner that the lead for the forward motion shall vary as little as possible. The following data are given:—the eccentricity $r = 0 \cdot 064^{m}$ ($2 \cdot 56''$), length of the eccentric-rods $l = 1 \cdot 560^{m}$ ($60''$), width of steam-ports $a = 0 \cdot 027^{m}$ ($1 \cdot 06''$). The lead for the highest grade is to be $v = 5^{mm}$ ($\cdot 196''$); the point K, at which the sliding-block stands at the last grade, is to be distant from the dead point as much as $c_1 = \mathrm{I\,K} = \frac{2}{3} c$ (link as in Fig. 7, Pl. II.), the half-length of the link $= \mathrm{C\,I} = c = 0 \cdot 21^{m}$ ($8 \cdot 26''$), and hence $c_1 = 0 \cdot 14^{m}$ ($5 \cdot 5''$).

Moreover let the valve-stem and piston-rod be parallel, and the centre lines of the two eccentrics form the angle $\mathrm{D\,O\,D_1} = 151\frac{1}{2}^{\circ}$, and let the inside lap be

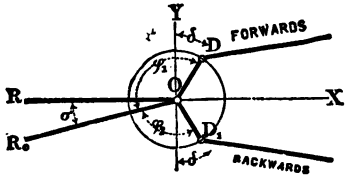
$$i = 4 \cdot 3^{mm} \ (\cdot 159'').$$

Assuming four grades of expansion, there are to be determined the outside lap and the whole distribution of the steam for the

forward motion, but first of all the best position for the crank is to be ascertained.

Solution.—First assume a crank which bisects the angle D O D_1 (Fig. 27). With regard to this crank, we have the angle

FIG. 27.



$$Y O D = \delta = \frac{151\frac{1}{2}^\circ}{2} - 90^\circ = 14\tfrac{1}{4}^\circ.$$

Next calculate the co-ordinates of the centre of the valve circle for the last grade of expansion, by substituting c_1 for u and $14\frac{1}{4}^\circ$ for δ in the known formulas for open rods,

$$a = \frac{1}{2} A = \frac{1}{2} r \left(\sin \delta + \frac{c^2 - u^2}{c\,l} \cos \delta\right),$$

$$b = \frac{1}{2} B = \frac{1}{2} \frac{r\,u}{c} \cos \delta,$$

and thus get

$$a = 0 \cdot 0102^{\text{m}}\ (0 \cdot 4''),$$

$$b = 0 \cdot 0207^{\text{m}}\ (0 \cdot 79'').$$

Now calculate the position of the centre of the valve circle for the dead point, by putting in the above formulas 0 for u, then

$$a_0 = 0 \cdot 0120^{\text{m}}\ (0 \cdot 47''),$$

and

$$b_0 = 0.$$

Now make in Fig. 11, Pl. III., $O B_4 = a = 0 \cdot 0102$, $B_4 C_4 = b = 0 \cdot 0207$, and $O B_0 = a_0 = 0 \cdot 0120$; describe from C_4 and B_0 the two valve circles; these cross each other at Q. Now draw O Q; the angle $Q O X = R_0 O R = \sigma = 5^\circ$ is the angular distance at which the actual crank in the present valve-gear should follow the imaginary one. Therefore if we draw in Fig. 27, the line $O R_0$ so that it forms with O R the angle $\sigma = 5^\circ$, then $R_0 O D = \phi_1 = 90^\circ + \delta + \sigma = 109\frac{1}{4}^\circ$ is the angle which the crank must make with the centre line of the forward eccentric, and $R_0 O D_1 = \phi_2 = 90^\circ + \delta - \sigma = 99\frac{1}{4}^\circ$ the angle which the crank must make with the back eccentric.

As the crank is on its dead point when it is on the line O X, the angle of advance of the forward eccentric is

$$\delta_1 = \phi_1 - 90^\circ = 19\tfrac{1}{4}^\circ,$$

and of the back eccentric

$$\delta_2 = \phi_2 - 90^\circ = 9\tfrac{1}{4}^\circ.$$

The principal part of the problem is therefore solved, and in a simpler manner than was to be expected when we consider the movement of the link, which has always been regarded as very complicated. The angle σ could also have been obtained by calculation; as the result of this calculation is extremely simple, we will now give it.

The formula for the movement of the valve (with open rods) is

$$\xi = r\left(\sin\delta + \frac{c^2 - u^2}{c\,l}\cos\delta\right)\cos\omega + \frac{u\,r}{c}\cos\delta\sin\omega.$$

Now in Fig. 11, Pl. III., O Q is nothing but the movement of the valve corresponding to the angular movement σ; if we, therefore, substitute σ for ω, and at the same time, for the last grade, substitute c_1 for u, and for the dead point 0 for u, then in one case we have

$$\mathrm{O\,Q} = r\left(\sin\delta + \frac{c^2 - c_1^2}{c\,l}\cos\delta\right)\cos\sigma + \frac{c_1\,r}{c}\cos\delta\sin\sigma,$$

and in the other case

$$\mathrm{O\,Q} = r\left(\sin\delta + \frac{c^2}{c\,l}\cos\delta\right)\cos\sigma,$$

Equating both expressions for O Q, we get, after reduction,

$$\tan\sigma = \frac{c_1}{l}.$$

This simple expression shows that the angular distance of the actual crank behind the imaginary one depends only *upon c_1 and the length l of the eccentric-rod.* The smaller c_1 and the larger l, the less the position of the crank has to be altered, and the more the values

of the two different angles of advance approach each other. This is why equal angles of advance are still used in many link-motions; for in these gears c_1 is so small and l so large, that the changing of the position of the crank is not necessary, as the lead varies but very little, even with equal angles of advance. This is, for example, the case with the locomotive of the North-East Railway of Switzerland, from which the dimensions of the preceding example were taken; the outside lap only has been taken different, as the example was for different angles of advance, whilst in this locomotive equal angles of advance were used. It was shown above that the distribution of the steam is not much improved by altering the position of the crank, it would therefore be better in designing valve-gears, to make c_1 as small as possible and to use long eccentric-rods, for the lead will then be less variable. If the engine does not allow of the use of long eccentric-rods, the employment of different angles of advance will always be an excellent way of equalizing the lead; of course, this is at the expense of correctness during the backward motion of the engine. Moreover, if we substitute the values of c_1 and l in the formula $\tan \sigma = \frac{c_1}{l}$, we get

$$\sigma = 5^\circ\ 8',$$

the same as the value given by the diagram. But the formula has the great advantage, that σ may be already determined when nothing else is known of a link-motion but the length of the eccentric-rods and the distance of the block from the dead point for the last grade of expansion. The formula gives at the same time a more simple way of determining the angle than the one given above. Construct a right-angled triangle with the sides c_1 and l, then the angle opposite to the side c_1 is the required one.

Calculation and construction show also another peculiarity when the position of the crank is altered in the manner stated, which may be briefly mentioned here. If there are given an even number of grades of expansion, or, including the dead point, an uneven number, the point of intersection of two valve circles which are equally remote from the dead point and the farthest grade, will always fall

for the forward motion on the line $O R_0$ (Fig. 8, Pl. II.); thus, as an example, in the case of six grades of expansion, the lead is equal for the dead point and the sixth grade; also equal for the first and fifth; it is next largest for the second and fourth, and is greatest (but only with open rods) for the central one, and thus in this case for the third. (See Diagram, Fig. 8, and Table on p. 96.)

Returning to the solution of the problem, we find the condition that the lead for the last grade is to be $v = 5^{mm}$ (·195″). Therefore, laying off upon $O R_0$, Fig. 11, Pl. III., from Q towards O, the value $V_6 Q = v = 5^{mm}$ (·196″), $O V_0 = 0 \cdot 019^m$ (·748″) will at once give the required outside lap. Now describe from O circles with $O V_0$ and the given inside lap $O W_0 = 4 \cdot 3^{mm}$ (·169″) as radii, then the whole distribution of the steam for the last grade is also known. Connecting the points of intersection $V_3 V_4 W_3 W_4$ with O, the principal positions of the crank are obtained; describe also from O any circle which represents the crank-pin circle, draw $L_0 L_1$ parallel to $R_0 R_1$, and make $L_1 L_0 = R_1 R_0$, then will $L_0 L_1$ represent the stroke of the piston. The line $O V_3$ cuts the crank-pin circle at R_3; dropping upon $L_0 L_1$ the perpendicular $R_3 L_3$, we get

$O R_3$ the position of the crank before the dead point, and L_3 the position of the piston before the end of the stroke, when the admission of the steam begins in cylinder end nearest the crank-shaft.

$O R_4$ is the position of the crank and L_4 that of the piston at the end of the admission of the steam, i.e. at the beginning of the expansion; $L_0 L_4 : L_0 L_1$ is the ratio of expansion with the last grade.

$O R_5$ is the position of the crank before the dead point, and $L_0 L_5$ the distance of the piston from the end of its stroke, when the release of the steam begins in the cylinder end farthest from crank-shaft. Finally, $O R_6$ is the position of the crank and L_6 is the position of the piston at the close of the exhaust from cylinder end farthest from crank-shaft, i.e. at the beginning of the compression; $L_0 L_6 : L_0 L_1$ is the ratio of compression.

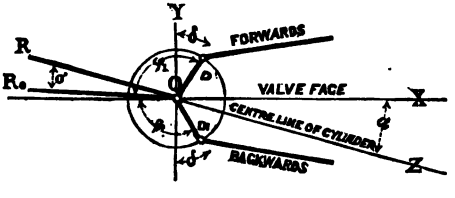
The principal positions for the other grades of expansion are

obtained in the same manner; first, the co-ordinates of the centres of the valve circles are calculated according to the known method, next the points of intersections of the valve circles with the lap circles are determined, &c.

If we wish to examine the distribution of the steam for the backward motion, which with different angles of advance is different from that of the forward motion, we must construct a diagram like that shown in Fig. 9, Pl. II., and must suppose the crank to turn in the opposite direction, i. e. in the direction of the arrow. But it must not be forgotten that O R_0 represents the position of the crank at its dead point, and that all angles must be measured from that position; with equal angles of advance, however, we estimate angles from the position O R.

We have expressly supposed, in the above example, that the centre line of the cylinder coincides with the direction of the valve-face; but if that is not the case, and if, moreover, the two directions O Z and O Y (Fig. 28) form an angle X O Z = a, we must proceed as follows:

Fig. 28.



Construct the diagram for equal angles of advance, and draw, as in Fig. 28, the two eccentricities O D and O D_1, so that they form with the line O Y, which is vertical to the direction of the valve-face, equal angles δ. The position of the eccentricities is thus obtained for the moment when the crank passes through the dead point, i. e. occupies the position O R.

But if we wish to employ different angles of advance in order to obtain as constant a lead as possible, the crank is set *back* to O R_0 as much as the angle σ, an angle deduced either from the diagram or by calculation.

It thus follows that the angle which the crank forms with the eccentricity of the forward eccentric is

$$R_0 O D = \phi_1 = 90^\circ + \delta - a + \sigma,$$

and the angle which the crank forms with the eccentricity of the back eccentric is

$$R_0 O D_1 = \phi_2 = 90^\circ + \delta + \alpha - \sigma,$$

as will be at once seen from Fig. 28. But these formulas are only correct for *Stephenson's* valve-gear with open rods and *when the valve-rod is moved directly by the link,* as in Fig. 5, Pl. II. But if the link acts upon a bell-crank lever, of which the arm connected with the sliding-block of the link has the length a, and the arm that moves the valve-rod the length b, then: 1st, the crank $O R_0$ (Fig. 28) must be reversed or set back 180°; and 2nd, in all formulas given above for *Stephenson's* valve-motion, the factor $\frac{b}{a} r$ must be substituted for the eccentricity r.

If, finally, the rods are crossed, then in accordance with previous propositions, the positions of the centres of the valve circles must be calculated according to the formulas:

$$a = \frac{1}{2} A = \frac{1}{2} r \left(\sin \delta - \frac{c^2 - u^2}{c\,l} \cos \delta \right),$$

$$b = \frac{1}{2} B = \frac{1}{2} \frac{r\,u}{c} \cos \delta,$$

from which a diagram like Fig. 10, Pl. II., is obtained.

The angle through which the crank must be turned in order to get as constant a lead as possible, may be obtained from the diagram in the same manner as in the valve-gear with open rods, but with the exception that the crank must in this case be set *forwards* the angular distance σ, i. e. set towards the forward eccentric.

The "Missing Quantity" in the Formula for Stephenson's Link-motion.

Equations (I^a) and (II^a), pp. 73 and 74, give for the movement of the valve the more exact expression

$$\xi = r \left(\sin \delta \pm \frac{c^2 - u^2}{c\,l} \cos \delta \right) \cos \omega \pm \frac{u\,r}{c} \cos \delta \sin \omega$$
$$+ \frac{r^2}{2\,l} \left(\cos 2\,\delta \sin \omega \pm \frac{u}{c} \sin 2\,\delta \cos \omega \right) \sin \omega.$$

The valve only moves symmetrically to both sides of a fixed point when the third quantity, the so-called "missing quantity," disappears, or, as it cannot ever disappear entirely, when it is as small as possible. But this last is always the case when the eccentric-rods are long and the eccentricities are small. In designing the link-motion, it is therefore of great importance to fulfil these conditions. The valve should always be set so that this "missing quantity" influences the movement of the valve most at those positions of the crank at which a little difference from the results of the diagram has the least effect upon the admission or release of the steam; this quantity has to be, therefore, especially small near the dead points, where the ports are opened least. The latter condition is fulfilled when the valve is set to give equal lead, i. e. set so that the ports are equally opened when the crank passes through one or the other dead point. Our formulas are obtained under the last supposition, and the fact that the "missing quantity" disappears in the formula for the movement of the valve when $\omega = 0$ or $180°$, and that the exact value for ξ is really obtained from the diagram, shows as a necessary consequence that the required condition has been fulfilled. As this is the case for all positions of the link, the result follows that setting or adjustment of the valve is only required for one grade of expansion; if for this grade an equal lead is obtained, it will be equal on both sides of the valve for all the other positions of the link. It is of no practical use to further investigate the influence of this quantity. If it is, however, necessary to employ very short eccentric-rods,* then, to obtain exact information about the movement of the valve, it is best to construct curves instead of valve circles; these curves are obtained by taking the "missing quantity" into account by substituting for ω in the above formulas (II^a and II^d, pp. 73 and 74) different values successively, and calculating ξ and laying it off from O upon the direction of the crank. If these points are connected, we obtain in place of the valve circles a looped curve, which gives in connection with the lap circles the chief positions of the crank, or the distribu-

* In this case it is desirable that the laps, especially the inside one, be not taken too small, else the release will not take place with sufficient regularity.

tion of the steam; the differences between these results and those of the circle diagram will then show whether the dimensions chosen for link-motion produce an admissible distribution of the steam.

CHAPTER II.

GOOCH'S LINK-MOTION.

Description of the Valve-gear.

GOOCH's link-motion belongs to that class of motions with variable expansion which has a constant lead for all grades of expansion. Fig. 12, Pl. III., shows in general the arrangement of the different parts of this construction. The two eccentrics D and D_1 are fastened to the axle O and govern the eccentric-rods B C and B_1 C_1, which are connected at their ends with the link C C_1. The link is also curved to a certain radius and has a slot formed in it, but its convex side is turned towards the axle. The link cannot be raised or lowered, as in *Stephenson's* link-motion, but at the dead point J it is fastened to a link which swings about the fixed point L. The point J therefore swings back and forth in an arc during each revolution of the axle; but as the radius of this arc is always very large, we may suppose, without committing a great error, that the point J oscillates in the direction of the stroke.

The sliding-block K, fastened to the radius-rod B_1 K, can move up and down in the slot of the link, while the other end B_1 is connected with the valve-stem, so that this point B_1 moves in the direction of the valve stroke. The raising or lowering of the radius-rod is effected by the hanger S T, which is carried by a bell-crank lever that is moved by a hand-lever, in the same manner as in *Stephenson's* link-motion. Figs. 29 and 30 represent the different parts of the valve-gear by lines only; the former is for *open*, the latter for *crossed rods*. In the following we designate the different parts in the same manner as before: O D = O D_1 = r are the eccentricities, which here *always* form with the normal line O Y the equal angles of advance

$Y\,O\,D_0 = Y_1\,O\,D'_0 = \delta$; the length of the eccentric-rods $D\,C = D_1\,C_1$ is again l; the length of the radius-rod $K\,B_1 = l_1$; the length of the valve-stem measured to the centre of the valve is $B_1\,B = l_2$; the half-length of the link $J\,C = J\,C_1 = c$, and the variable distance of the sliding-block from the dead point is $J\,K = u$.

FIG. 29.

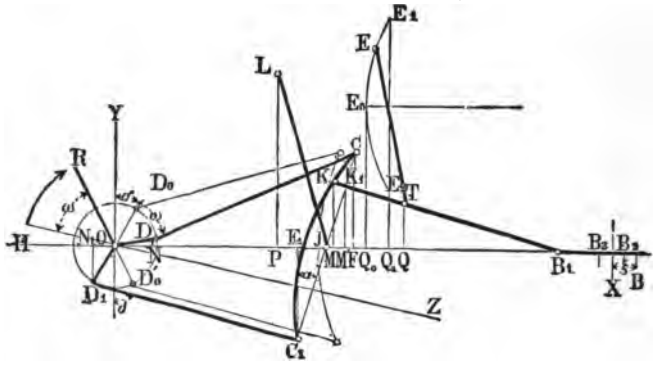
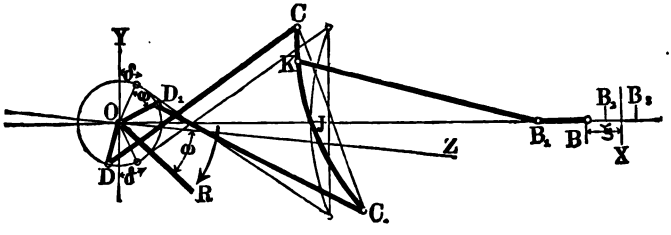


FIG. 30.



THEORY OF GOOCH'S LINK-MOTION.

(*a*). *Determination of the Valve Movement.*

The fine lines in Figs. 29 and 30 represent the positions of the different parts of the arrangement when the crank stands at one dead point, say in the direction O Z; if the crank is now turned from this position through the angle ω, the different parts will then arrive at the positions represented by the heavy lines.

Now the problem is to determine the distance ξ of the valve from its centre of motion for this angle ω, and for any position of the sliding-block K or any value u.

We shall at first assume that *open* eccentric-rods are used (Fig. 29), and to begin with, shall assume the link CC_1 to be straight.

The angle a, which the chord of the link forms with a line vertical to the stroke for any angular movement, has already been determined on p. 70, equation (17), under the supposition that the dead point J of the link oscillates in the line of the stroke, as is the case here. We found

$$\sin a = \frac{r}{c} \cos \delta \sin \omega - \frac{r}{l} \sin \delta \sin \omega + \frac{r^2}{4cl} [\cos^2 (\delta - \omega) - \cos^2 (\delta + \omega)].$$

If we drop from the point C upon the direction of the stroke the perpendicular CF, we have

$$OF = ON + NF = ON + \sqrt{DC^2 - (CF - DN)^2},$$

or according to the above notation,

$$OF = r \sin (\delta + \omega) + \sqrt{l^2 - [c \cos a - r \cos (\delta + \omega)]^2}.$$

If we develop the expression under the radical into a series, neglect the quantities which have l^2 in the denominator, and put $\cos a = 1$,—for a has always a very small value,—then,

$$OF = r \sin (\delta + \omega) + l - \frac{c^2}{2l} + \frac{r c \cos (\delta + \omega)}{l} - \frac{r^2 \cos^2 (\delta + \omega)}{2l}. \quad (41)$$

Now, the distance of the centre B of the valve from the centre O of the axle is

$$OB = OM + MB_1 + BB_1,$$

or

$$OB = OF - M'F - MM' + MB_1 + BB_1.$$

OF is known from equation (41), and

$$M'F = (c - u) \sin a,$$

where $\sin a$ is given by equation (17). $M M' = K K_1$ or, when the radius of the link $= \rho$,

$$M M' = \frac{c^2}{2\rho} - \frac{u^2}{2\rho},$$

$$M B_1 = \sqrt{B_1 K^2 - K M^2} = \sqrt{l_1^2 - u^2} = l_1 - \frac{u^2}{2 l_1};$$

for l_1 is always large in proportion to u.

Finally $B B_1 = l_2$, and therefore, after substitution and reduction,

$$\begin{aligned} O B = {} & r\left(\sin\delta + \frac{c}{l}\cos\delta\right)\cos\omega + \frac{u r}{c}\left(\cos\delta - \frac{c}{l}\sin\delta\right)\sin\omega \\ & + l - \frac{c^2}{2 l} - \frac{c^2}{2\rho} + \frac{u^2}{2\rho} + l_1 - \frac{u^2}{2 l_1} + l_2 \\ & - \frac{r^2}{4 c l}\left[(c+u)\cos^2(\delta+\omega) + (c-u)\cos^2(\delta-\omega)\right]. \qquad (42) \end{aligned}$$

Now the valve has to move symmetrically backwards and forwards on both sides of a point X, whose distance from O we must next determine; we also assume that the valve is adjusted so as to give equal lead.

If the crank stands at the one dead point, then $\omega = 0$, and the distance of the centre of the valve from the centre of the axle will be

$$\begin{aligned} O B_2 = {} & r\left(\sin\delta + \frac{c}{l}\cos\delta\right) + l + l_1 + l_2 - \frac{c^2}{2 l} - \frac{c^2}{2\rho} \\ & + \frac{u^2}{2\rho} - \frac{u^2}{2 l_1} - \frac{r^2}{2 l}\cos^2\delta. \end{aligned}$$

For the second dead point, or for $\omega = 180°$ we get

$$\begin{aligned} O B_3 = {} & -r\left(\sin\delta + \frac{c}{l}\cos\delta\right) + l + l_1 + l_2 - \frac{c^2}{2 l} - \frac{c^2}{2\rho} \\ & + \frac{u^2}{2\rho} - \frac{u^2}{2 l_1} - \frac{r^2}{2 l}\cos^2\delta. \end{aligned}$$

The arithmetical mean of the two values gives the distance of the centre of motion X from the centre of the axle:—

$$O X = l + l_1 + l_2 - \frac{r^2}{2 l}\cos^2\delta - \frac{c^2}{2 l \rho}(l + \rho) + \frac{u^2}{2\rho l_1}(\rho - l_1). \qquad (43)$$

The centre of motion has to remain unaltered for all grades of expansion, i.e. for all values of u; but according to equation (43) that will only be the case when the last term, which contains u, is *nil*. This condition can and must be fulfilled in this valve-gear, for it is only necessary to make

$$\rho = l_1$$

i.e. the link of Gooch's link-motion should always be curved to an arc whose radius is equal to the length l of the radius rod.

Equation (43) then gives

$$\mathrm{O\,X} = l + l_1 + l_2 - \frac{r^2 \cos^2 \delta}{2\,l} - \frac{c^2}{2\,l\,l_1}(l + l_1). \qquad (44)$$

Finally, the movement of the valve from its central position or the movement ξ for the angular movement ω and for the grade of expansion u, is

$$\xi = \mathrm{O\,B} - \mathrm{O\,X};$$

or using equations (42) and (44), under the supposition that $l_1 = \rho$, and reducing, we get

$$\text{(III}^{\text{a}}\text{)} \quad \xi = r\left(\sin\delta + \frac{c}{l}\cos\delta\right)\cos\omega + \frac{u\,r}{c}\left(\cos\delta - \frac{c}{l}\sin\delta\right)\sin\omega$$
$$+ \frac{r^2}{2\,l}\left(\cos 2\,\delta \sin\omega + \frac{u}{c}\sin 2\,\delta\cos\omega\right)\sin\omega.$$

If, however, *crossed* rods (Fig. 30) are assumed, we must substitute negative values of c in the above formula, and at once get for this case

$$\text{(III}^{\text{b}}\text{)} \quad \xi = r\left(\sin\delta - \frac{c}{l}\cos\delta\right)\cos\omega - \frac{u\,r}{c}\left(\cos\delta + \frac{c}{l}\sin\delta\right)\sin\omega$$
$$+ \frac{r^2}{2\,l}\left(\cos 2\,\delta \sin\omega - \frac{u}{c}\sin 2\,\delta\cos\omega\right)\sin\omega.$$

As l is always large in proportion to r, and as in both formulas the third quantity, "the missing quantity," in consequence of the assumed manner of adjusting the valve, is greatest at those points where it influences the distribution of the steam least, this quantity

may be neglected in both cases, and we get for the movement of the valve for open and crossed eccentric-rods

$$\text{(III)} \quad \xi = r\left(\sin\delta \pm \frac{c}{l}\cos\delta\right)\cos\omega \pm \frac{ur}{c}\left(\cos\delta \mp \frac{c}{l}\sin\delta\right)\sin\omega.$$

Again abbreviating, we have

$$r\left(\sin\delta \pm \frac{c}{l}\cos\delta\right) = \mathrm{A}, \tag{45}$$

$$\frac{ur}{c}\left(\cos\delta \mp \frac{c}{l}\sin\delta\right) = \mathrm{B}, \tag{46}$$

and then

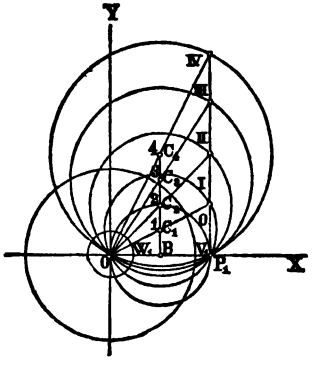
$$\xi = \mathrm{A}\cos\omega \pm \mathrm{B}\sin\omega. \tag{47}$$

The law of the valve movement is therefore the same as that already found for the simple valve-gear and *Stephenson's* link-motion.

(*b*). *The Curve of Centres.*

As we have found for the movement of the valve in *Gooch's* link-motion an equation of the same form as for *Stephenson's*, we may also here consider the movements of the valve for a certain position of the link as chords of a circle, the co-ordinates of whose centres are (Fig. 31):

Fig. 31.



$$\mathrm{O\,B} = a = \frac{\mathrm{A}}{2} = \frac{r}{2}\left(\sin\delta \pm \frac{c}{l}\cos\delta\right)$$

and

$$\mathrm{B\,C} = b = \frac{\mathrm{B}}{2} = \frac{ru}{2c}\left(\cos\delta \mp \frac{c}{l}\sin\delta\right).$$

There is therefore in this case also a valve circle corresponding to each grade of expansion, i. e. one for each value of u which is used.

The centres of all these circles will again lie on a curve, the curve of centres, whose law is to be determined.

In *Stephenson's* link-motion this curve is a parabola, but it is still simpler in the present link-motion; for as the formula for the abscissa a does not contain the value u, it follows that the centres of

all the valve circles have the same abscissa, and are thus situated in a perpendicular to the line of stroke O B; the "curve" of centres is a *straight* line BC_4 (Fig. 31), which is normal to the centre line of motion, and is distant from the centre of the axle for *open* eccentric-rods as much as

$$\frac{r}{2}\left(\sin\delta + \frac{c}{l}\cos\delta\right); \tag{48}$$

but for *crossed* rods as much as

$$\frac{r}{2}\left(\sin\delta - \frac{c}{l}\cos\delta\right). \tag{49}$$

We found on p. 75 the same expressions for *Stephenson's* link-motion, but there they represent the distance from the centre of the axle of the vertices of those parabolas which form the curve of centres. The close relation between *Gooch's* link-motion and that by *Stephenson* follows also from the comparison of the "missing quantities" of the two motions; this quantity is in both constructions

$$\frac{r^2}{2l}\left(\cos 2\delta \sin\omega \pm \frac{u}{c}\sin 2\delta\cos\omega\right)\sin\omega.$$

The remarks previously made concerning this quantity are correct for the present valve-motion, and it is therefore unnecessary to repeat them.

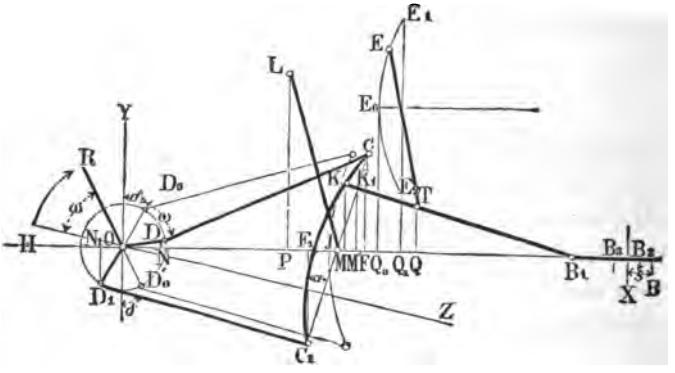
(c.) *The Suspension of the Link and of the Radius-rod.*

There are in *Gooch's* link-motion, two suspension-links: the one L J (Fig. 32) carries the link and swings round a fixed pin at L, while its lower end is connected with the link. The latter point of connection is in practice generally placed behind the curve towards the axle. Experiments with a model have shown that this causes irregularities in the movement of the link, in consequence of which the slide-block K during the movement slips up and down in the slot of the link, but that these irregularities are very much reduced if the point of suspension of the link is placed nearer to the centre J of the chord.

It is very difficult to prove the correctness of this remark in a mathematical way, but it may be inferred from the above theoretical

investigations. We there always assumed, that the centre J of the chord of the link moves back and forth on the line of stroke, and under this supposition found that the end B_1 of the radius-rod B_1 K moves symmetrically back and forth on both sides of a point in a manner suitable for governing the movements of a steam-valve.

FIG. 32.



From this would follow, that in practice care should be taken that the centre of the chord move as exactly as possible in the straight line O B. This condition is fulfilled when the link is suspended at this point J.

Now, the next thing is to ascertain the abscissa of the fixed point L; of course, the ordinate is the length of the suspension-rod itself, and for this the only rule we have is, to make it as long as possible.

We may use, for determining the abscissa of L, equations already given.

Equation (30) gives the abscissa of the centre of the chord for that moment when it passes through the centre of its arc of oscillation, or through its centre of motion, and this equation is

$$x = l - \frac{c^2 + u^2}{2\,l},$$

where the link is supposed to be lowered as much as u. The question in the present case is the same, but here the link is always so placed that the point J moves back and forth almost exactly in

the direction of the stroke, we therefore have $u=0$, and the abscissa of the centre of oscillation is consequently more simple, namely,

$$x = l - \frac{c^2}{2\,l}. \qquad (50)$$

Now, as the suspension-rod has to be in a vertical position when the point J passes through its centre of oscillation, *the above formula gives also the abscissa of the point of suspension* L, measured from the centre O of the axle on the line of stroke; it is therefore only necessary to subtract the versed sine of the arc of the link from the length of the eccentric-rods.

The *second* suspension-rod E T carries the radius-rod. As the latter has to be raised or lowered, the point of suspension E must be movable. The question now is, in what curve this movement must take place in order that the sliding-block K may alter its position in the link as little as possible; the problem is to arrange the suspension in such a manner, that the vertical movement of the point T is a minimum. As T is supposed to move in an arc of the radius E T, this vertical movement will be very small when E T is large and the chord is parallel to the line of stroke O B; it is unnecessary to say much here concerning the first condition, and we shall therefore at once consider the second one. For that purpose we determine next the horizontal movement of the point T for any grade u, that is, the variable abscissa O Q. We have

$$\mathrm{O\,Q} = \mathrm{O\,B} - \mathrm{B\,B_1} - \mathrm{B_1\,Q}.$$

But O B is given by equation (42). Neglecting the last term, which contains the very small factor $\frac{r^2}{4\,c\,l}$, and substituting l_1 for ρ, we obtain

$$\mathrm{O\,B} = r\left(\sin\delta + \frac{c}{l}\cos\delta\right)\cos\omega + \frac{u\,r}{c}\left(\cos\delta - \frac{c}{l}\sin\delta\right)\sin\omega$$
$$+ l + l_1 + l_2 - \frac{c^2}{2\,l\,l_1}(l + l_1).$$

We also have $\mathrm{B\,B_1} = l_2$ and $\mathrm{B_1\,Q} = \sqrt{\mathrm{B_1\,T^2} - \mathrm{Q\,T^2}}$; representing $\mathrm{B_1\,T}$ by l_0, and assuming approximately that $\mathrm{K\,M} = u$, then

$$\mathrm{Q\,T} = \frac{l_0}{l_1}\,u,$$

hence

$$B_1 Q = \sqrt{l_0^2 - \frac{l_0^2 u^2}{l_1^2}},$$

or approximately, for l_1 is always very large in proportion to u,

$$B_1 Q = l_0 \left(1 - \frac{u^2}{2 l_1^2}\right).$$

We thus get

$$O Q = r\left(\sin \delta + \frac{c}{l} \cos \delta\right) \cos \omega + \frac{u r}{c}\left(\cos \delta - \frac{c}{l} \sin \delta\right) \sin \omega$$
$$+ l + l_1 - l_0 + \frac{l_0 u^2}{2 l_1^2} - \frac{c^2 (l + l_1)}{2 l l_1}.$$

Substituting for ω in this equation, first 0, and then 180°, and taking the mean x of the two values obtained, we get

$$x = l + l_1 - l_0 - \frac{c^2 (l + l_1)}{2 l l_1} + \frac{l_0 u^2}{2 l_1^2}, \tag{51}$$

and this *is the abscissa of the point of suspension* E for the corresponding position of the radius-rod.

If l_3 is the length of the suspension-rod, the ordinate is approximately

$$y = l_3 + \frac{l_0}{l_1} u. \tag{52}$$

If the sliding-block is at the dead point of the link, $u = 0$, and therefore the co-ordinates for this position are

$$O Q_0 = l + l_1 - l_0 - \frac{c^2 (l + l_1)}{2 l l_1} \quad \text{and} \quad Q_0 E_0 = l_3. \tag{53}$$

The point E_0 is found by a simple calculation. If the radius-rod occupies the highest or lowest position, then $u = + c$ or $- c$, for the sliding-block K of this kind of valve-gears can always be shifted into positions which coincide with the centre lines of the eccentric-rods, as to C and C_1. Substituting these values of u in equations (51) and (52), we get next, for the two extreme positions E_1 and E_2 of the point of suspension, the abscissa

$$O Q_1 = l + l_1 - l_0 - \frac{c^2 (l + l_1)}{2 l l_1} + \frac{l_0 c^2}{2 l_1^2}, \tag{54}$$

the ordinates, however, are

$$\left.\begin{aligned} Q_1 E_1 &= l_3 + \frac{l_0}{l_1} c \\ \text{and} \qquad & \\ Q_1 E_2 &= l_3 - \frac{l_0}{l_1} c. \end{aligned}\right\} \qquad (55)$$

The point of suspension should therefore be moved in an arc which passes through these three points $E_1 E_0 E_2$. But the combination of the two equations (51) and (52) gives again the peculiar result, that the curve which passes through the above points is a *parabola the parameter of which is* $= 2 l_1$, *or equal to double the length of the radius-rod.* Now, as l_1 is always very large, the short part of the parabola, which lies near the vertex, may be replaced by an arc of the radius l_1, *i. e. by radius equal to the length of the radius-rod.*

This radius is taken much smaller in practice, and after the positions $E_1 E_0 E_2$ are laid off according to the co-ordinates above calculated, these points are moved so far parallel to the line of motion, that the arc passing through them has the desired radius. But according to the preceding theory, the irregularities in the movement of the point K will increase the smaller this radius is made.

The preceding theory of the suspension of the link is, moreover, applicable not only to *open* eccentric-rods, but to *crossed* ones as well.

Practical Application of the Diagram.

We can refer in what follows to different facts which have already been stated during the investigation of the diagram of the simple valve-gear and of *Stephenson's* link-motion; the same notation has been taken for the present valve-gear as for the preceding ones. In order to examine the distribution of the steam effected by *Gooch's* link-motion, it is necessary to draw, as before, a valve circle for each grade of expansion, i. e. for each value of u; in other words, to calculate the co-ordinates for its centre, and lay them off on the drawing.

The theory gives for the movement ξ of the valve the formula:

$$\text{(III.)} \quad \xi = r\left(\sin\delta \pm \frac{c}{l}\cos\delta\right)\cos\omega \pm \frac{u\,r}{c}\left(\cos\delta \mp \frac{c}{l}\sin\delta\right)\sin\omega,$$

in which the *upper* sign is to be taken for open eccentric-rods, and the *lower* sign for crossed eccentric-rods.

The co-ordinates for the centres of the valve circles are then

$$\text{O B} = a = \frac{r}{2}\left(\sin\delta \pm \frac{c}{l}\cos\delta\right) \tag{56}$$

$$\text{B C} = b = \frac{u\,r}{2\,c}\left(\cos\delta \mp \frac{c}{l}\sin\delta\right); \tag{57}$$

these values may easily be calculated, when the values of r, δ, c, l, and u are known. For a given valve-gear u is the only variable quantity, and may be expressed in parts of c. As already stated, the valve-gears at present under discussion always allow the sliding-block K to occupy positions which coincide with the centre lines of the eccentric-rods, and thus the greatest value of u is equal to $+c$ or $-c$. The half-length c of the link is now again divided from the dead point each way, into a certain number of equal parts or grades of expansion; thus if we have n parts and if the sliding-block is at the mth part, then

$$u = \frac{m}{n} \cdot c.$$

By substituting this value in the above equations (56) and (57), the co-ordinates of the centres of the valve circles for the corresponding grades will be obtained; the circle may then be drawn, and in connection with the lap circles will show the distribution of the steam for the corresponding grade in the manner already explained. It will be advantageous to show at once by an example the manner in which all questions relating to the distribution of the steam by an existing or assumed valve-gear may be answered by the diagram.

Problem.—The following measurements are taken from a link-motion on Gooch's system:—Eccentricity $r = 0{\cdot}060^{\text{m}}$ $(2{\cdot}36'')$; angle of advance $\delta = 20°$; half-length of the link $c = 0{\cdot}150^{\text{m}}$ $(6'')$;

length of the eccentric-rods $l = 1 \cdot 2^m$ (48″); the outside lap $e = 0 \cdot 023^m$ (0·91″); the inside lap $i = 0 \cdot 006^m$ (0·23″); there are four grades of expansion for the forward gear and the same number for back gear.

How is the distribution of the steam effected?

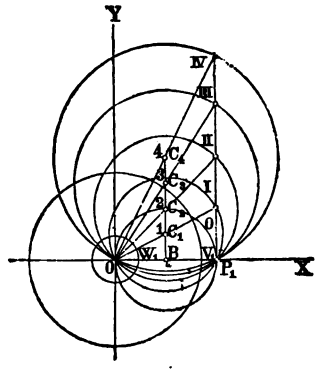
Calculate first the co-ordinates O B and B C_4 of the centre of the valve circle C_4 for the highest grade of expansion, thus for $u = c$ (Fig. 33). The formulas

$$O\,B = \frac{r}{2}\left(\sin\delta + \frac{c}{l}\cos\delta\right)$$

$$B\,C_4 = \frac{r}{2}\left(\cos\delta - \frac{c}{l}\sin\delta\right)$$

give, if the above values are used,

FIG. 33.



$$O\,B = 0 \cdot 014^m\ (0 \cdot 56'') \quad \text{and} \quad B\,C_4 = 0 \cdot 027^m\ (1 \cdot 06'').$$

If these two values are marked from O on the axis O X, as shown in Fig. 33, the centre C_4 of the valve circle, corresponding to the last or fourth grade of expansion, is obtained. Describe from C_4 this circle with C_4 O as radius.

Next simply divide the ordinate B C_4 into as many parts as there are grades, in this case into four. The points of division C_1, C_2, and C_3, then give at once the centres of the valve circles belonging to the different grades, and these circles are described from C_1, C_2, and C_3 with C_1 O, C_2 O, and C_3 O as radii. If, finally, circles are described from O with the outside lap O $V_1 = e = 0 \cdot 023^m$ and the inside lap O $W_1 = i = 0 \cdot 006$ as radii, the diagram will be complete. How, for each grade of expansion, the chief positions of the crank, for instance the beginning of the admission, of the exhaust, of the expansion, of the compression, the greatest opening of the ports, &c., are to be obtained from the points of intersection of the valve circles with the lap circles, has been already explained, and it will therefore be unnecessary to repeat it.

We need therefore only call attention to one peculiarity of this

valve-gear; it will be seen that every valve circle crosses the axis of the abscissa at the same point P_1; but now it is known that $V_1 P_1$ is the opening of the port for the admission, and $W_1 P_1$ is the opening of the port for the release at the beginning of the stroke, or the outside and inside lead respectively; hence it will be seen, that the lead is the same for all grades. The measurements from the figure, which is drawn half-size, give

$$\text{Outside lead } V_1 P_1 = 0 \cdot 005^m \, (0 \cdot 2''),$$
$$\text{Inside } \quad ,, \quad W_1 P_1 = 0 \cdot 022^m \, (0 \cdot 86'').$$

For the dead point of the link $u = 0$, the abscissa of the centre of its valve circle,

$$O B = \frac{r}{2}\left(\sin \delta + \frac{c}{l} \cos \delta\right);$$

the ordinate, however, is *nil*, and the centre therefore coincides with B.

We obtained the same result for the dead point of Stephenson's link, and it follows, that with equal dimensions of the two valve-gears, the distribution of the steam by the dead point is exactly the same. The remarks on p. 90 are also correct for the present case. It is generally considered to be a great advantage, that in the case of the present valve-gear, the lead for open eccentric-rods as well as for crossed ones, is constant, and this valve-gear is, therefore, considered to be better than that by *Stephenson*, in which the lead, as we have explained, increases with open rods and decreases with crossed ones, the higher the grade of expansion. But from the results obtained above, we are obliged to declare the valve-gear by *Stephenson* to be the better one. *Gooch's* valve-gear always requires a great distance between the driving axle and the cylinder, for the long radius-rod has to be placed between the link and valve-rod; hence this arrangement will, especially in locomotive engines, be applied but seldom. If a constant lead is really necessary, then, as we have shown, *Stephenson's* valve-gear may be so constructed that the lead in this case also becomes constant. Take a short link, long eccentric-rods, and different angles of advance, and the small variations of the lead which may occur with such an

arrangement of *Stephenson's* valve-gear cannot possibly affect disadvantageously the action of the steam in the cylinder.

A second problem would be to design and construct for a steam-engine, say for a locomotive, a valve-gear on *Gooch's* system by means of a diagram. There is no difficulty in doing this according to the methods which have been explained above. Certain parts, as the length of the eccentric-rods and the length of the link, are already generally determined by the whole arrangement of the engine. If we have the choice, then, we start on the principle of making the eccentric-rods as long, and the link as short, as possible. We next choose the angles of advance between 10° and 30°, the eccentricities between about $0 \cdot 050^m$ (2″) and $0 \cdot 080^m$ (3·2″) in such manner that the greater eccentricity corresponds to a smaller angle of advance. We may now at once draw the diagram for the last grade of expansion, and obtain by $O P_1$ (Fig. 33) the movement of the valve from its central position at the commencement of the stroke of the piston. If we now lay off from P_1 towards O the distance $P_1 V_1$ equal to the outside lead, amounting generally from $0 \cdot 004^m$ (0·16″) to $0 \cdot 007^{m1}$ (0·28″), we get $O V_1$ the outside lap, which is generally between $0 \cdot 020^m$ (0·8″) and $0 \cdot 040^m$ (1·6″). If the diagram shows this lap to be too large, a smaller angle of advance or a smaller eccentricity must be taken. The inside lap is generally taken between $0 \cdot 001^m$ (0·04″) and $0 \cdot 007^m$ (0·28″).

The above rules are also correct for Stephenson's valve-gear, except that in such a case the outside lap has to be chosen with regard to the variability of the outside lead. We, therefore, draw the valve circle for the dead point, and examine whether the lead for the inner grades of expansion with open rods is too large, or whether with crossed rods under the chosen proportions, it is too small.

Finally, for the suspension of the link and of the radius-rod, we refer to the results of the theoretical investigations.

CHAPTER III.

ALLAN'S LINK-MOTION.

Description of the Valve-gear.

IN *Stephenson's* link-motion, as well as in *Gooch's*, the link is curved to an arc, which in the former has a radius equal to the length of the eccentric-rod, and in the latter equal to the length of the radius-rod. We have already proved that this must be so if the valve is to swing symmetrically on each side of a point that remains fixed for all grades of expansion, a condition of things required for a regular and correct distribution of the steam.

But the construction of a curved link with large radius formerly involved great practical difficulties, and the often-repeated question, whether the curved link could not be replaced by a straight one, was therefore quite natural. But the arrangement of the different parts of *Stephenson's* and *Gooch's* link-motion will not allow this, as may be easily shown theoretically.

If in equations (19), p. 72, and (43), p. 110, the radius ρ of the curved link be taken infinitely large, two equations will be obtained, the first one for *Stephenson's* link-motion, and the second one for *Gooch's*, which give the distance of the centre of motion of the movement of the valve from the centre of the axle. These equations will show that, under the given supposition, the centre of motion of the valve *moves farther and farther* from the centre of the axle the more the link (*Stephenson's* gear) or the radius-rod (*Gooch's* gear) is *raised or lowered.* But the consequence of such an alteration of the centre of motion would be, that the admission and the release of the steam, in fact the distribution of the steam for the forward stroke would be quite different from that for the return stroke.

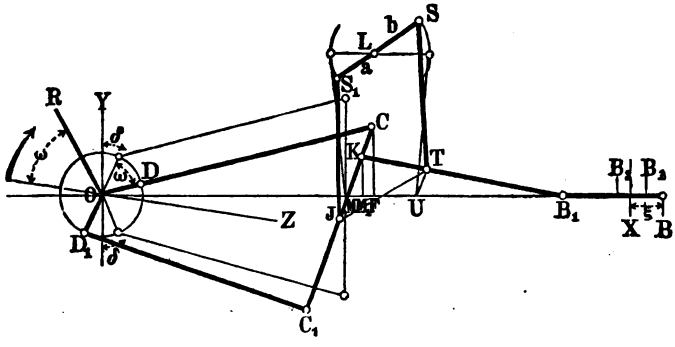
Viewed in this light, *Allan's* invention—an invention which was also made a little later by *Trick* in Germany, undoubtedly independent of *Allan*—appears the more interesting. Both Allan and Trick employ a *straight* link, and attain their end perfectly, as theory will show, by the ingenious plan of raising or lowering the link and the

radius-rod of *Gooch's* gear *simultaneously*; i. e. the valve moves at *all* grades of expansion symmetrically on each side of a fixed point.

Fig. 13, Pl. III., gives a plan and elevation of *Allan's* link-motion.

The two eccentrics D and D_1, fastened to the axle O with equal angles of advance, govern the two eccentric-rods D C and D_1 C_1, which are connected at their farther ends C and C_1 to the link H H. The link has a straight slot, in which the sliding-block, connected with the end of the radius-rod B_1 K, can be moved up and down. The radius-rod is connected at its end B_1 with the guided valve-stem B_1 B. The link is carried at its dead point J by the lifting-link S_1 J, while the lifting-link S T is at its one end fastened to the radius-rod B_1 K; both lifting-links are connected at their other ends by a double lever S_1 L S, which is fixed to the reversing-shaft L. To the same shaft is fastened the lever N, which can be moved between the positions indicated by dotted lines, by means of the reversing-rod P. It will now be seen that turning the shaft L simultaneously lowers the link and raises the radius-rod and its sliding-block K; on the other hand, when the link rises, the radius-rod is lowered. The various dimensions of this link-motion must also be in certain proportion, if the movement of the valve is to be a correct one. Everything depends, as the theoretical investigations will show, upon the ratio of the two arms L S and S_1 L of the double lever S S_1,

Fig. 34.



For our investigations we shall use the sketch, Fig. 34, and retain for the different parts the notation previously employed. Thus,

as before, let r = the eccentricity $O D = O D_1$, δ = the angle of advance, l = the length of the eccentric-rods $C D = C_1 D_1$, c = the half-length $J C = J C_1$ of the link; l_1 = the length of the radius-rod $B_1 K$ and l_2 = that of the valve-rod measured to the centre of the valve B. Also let the distance between the connecting point T of the lifting-arm and the end B_1 of the radius-rod be l_0, and let the two arms $S_1 L$ and $S L$ be represented by a and b. Finally, let the distance of the sliding-block K from the dead point J of the link for any position of the latter be u, thus $J K = u$. The link crosses the line of motion O B at the point M; let the distance of this point from the dead point of the link be $J M = u_1$, and let the distance of the same point from the sliding-block K be $M K = u_2$; then

$$u = u_1 + u_2.$$

Theory of Allan's Link-motion.

(a.) *Determination of the Movement of the Valve.*

We shall commence here also by establishing the equation which gives the distance of the centre of the valve B from the centre O of the axle for a certain position of the link and for any angular movement ω of the crank; we shall use the notation given above, and a few equations which were found while developing the theory of *Stephenson's* valve-gear, as we in that case at first assumed a straight link. Let us first take the case of *open* eccentric-rods.

According to Fig. 34,

$$O B = O F - F M_1 + M_1 B_1 + B_1 B. \tag{58}$$

Some of the different quantities on the right-hand side of this equation are known, and the rest are easy to determine. If we substitute u_1 for u in equation (15), we get

$$O F = r \sin(\delta + \omega) + l - \frac{c^2}{2l} + \frac{c u_1}{l} - \frac{u_1^2}{2l} + \frac{(c - u_1) r \cos(\delta + \omega)}{l} - \frac{r^2 \cos^2(\delta + \omega)}{2l}. \tag{59}$$

Moreover

$$F M_1 = (c - u) \sin \alpha; \tag{60}$$

where, according to equation (16), p. 70,

$$\sin \alpha = \frac{r}{c} \cos \delta \sin \omega - \frac{r}{l} \sin \delta \sin \omega - \frac{u_1 r}{c l} \cos \delta \cos \omega$$
$$+ \frac{u_1}{l} + \frac{r^2}{4 c l} [\cos^2 (\delta - \omega) - \cos^2 (\delta + \omega)]. \qquad (61)$$

The value of $M_1 B_1$, according to Fig. 34, is

$$M_1 B_1 = \sqrt{\overline{B_1 K}^2 - \overline{K M_1}^2};$$

or, as $B_1 K = l_1$, and approximately $K M_1 = u_2$,

$$M_1 B_1 = \sqrt{l_1^2 - u_2^2};$$

or, since u_2 is small in proportion to l_1,

$$M_1 B_1 = l_1 - \frac{u_2^2}{2 l_1} = l_1 - \frac{(u - u_1)^2}{2 l_1}. \qquad (62)$$

And finally

$$B B_1 = l_2.$$

Substituting these different values in equation (58), we get, after sufficient reduction,

$$O B = r \left(\sin \delta + \frac{c^2 - u u_1}{c l} \cos \delta \right) \cos \omega$$
$$+ \frac{r u}{c} \left(\cos \delta - \frac{c (u - u_1)}{u l} \sin \delta \right) \sin \omega + l + l_1 + l_2$$
$$- \frac{c^2}{2 l} - \left[\frac{u_1^2}{2 l} - \frac{u u_1}{l} + \frac{(u - u_1)^2}{2 l_1} \right]$$
$$- \left[\frac{c - u}{c} \cos^2 (\delta - \omega) + \frac{c + u}{c} \cos^2 (\delta + \omega) \right]. \qquad (63)$$

If we substitute in this equation first 0 for ω, and next 180° for ω, and take the mean of the two values thus obtained, i. e. if we adjust the valve to give equal lead, the distance of the centre of motion X of the valve from the centre of the axle will be

$$O X = l + l_1 + l_2 - \frac{c^2}{2 l} - \frac{r^2 \cos^2 \delta}{2 l}$$
$$- \left[\frac{u_1^2}{2 l} - \frac{u u_1}{l} + \frac{(u - u_1)^2}{2 l_1} \right], \qquad (64)$$

but u and u_1 are variable values, and the valve-gear would be quite useless if the last quantity between the brackets could not be reduced to *nil*, i. e. if the centre of motion X could not be kept at the same place for every grade of expansion. But, fortunately, this may easily be done; let us try to ascertain the corresponding relation between u and u_1.

First, it is easy to see that the following proportion is correct, if in Fig. 34 we draw T U parallel to the link:

$$\mathrm{T\,U} : \mathrm{K\,M} = \mathrm{B_1\,T} : \mathrm{B_1\,K},$$

or, if $\mathrm{T\,U} = u_2$,

$$u_2 = \frac{l_0}{l_1} u_1. \tag{65}$$

We may then find the relation between u_2 and u_1. The lifting-links $\mathrm{S_1\,J}$ and S T are of equal length, and when K is at the dead point, the points J and T fall on the line of motion, and the lever $\mathrm{S\,S_1}$ is parallel to the latter. As the reversing-shaft is turned, the points J and T move in directions nearly vertical to O B, and under these suppositions it will be easy to prove that the following proportionate movement takes place:

$$\mathrm{T\,U} : \mathrm{J\,M} = \mathrm{S\,L} : \mathrm{S_1\,L},$$

or, using the known notation,

$$u_2 = \frac{b}{a} u_1. \tag{66}$$

The combination of this equation with equation (65) gives

$$u_2 = \frac{l_1}{l_0} \frac{b}{a} u_1.$$

Therefore

$$u_1 + u_2 = \left(1 + \frac{l_1}{l_0} \frac{b}{a}\right) u_1,$$

or, since $u_1 + u_2 = u$,

$$u = \left(1 + \frac{l_1}{l_0} \frac{b}{a}\right) u_1. \tag{67}$$

Placing the constant value,

$$1 + \frac{l_1}{l_0} \frac{b}{a} = n, \tag{68}$$

we get $u = n\,u_1$. The value of n may always be calculated for a given valve-gear when the ratio $b : a$ of the arms of the lever, and the lengths l_1 and l_0 are known.

After these preparations, we return again to formula (64). We found that with Allan's valve-gear the following condition must always be fulfilled:

$$\frac{u_1^2}{2\,l} - \frac{u\,u_1}{l} + \frac{(u - u_1)^2}{2\,l_1} = 0.$$

Now if we substitute in this $u = n\,u_1$, we get

$$\frac{1}{2\,l} - \frac{n}{l} + \frac{(n-1)^2}{2\,l_1} = 0,$$

and thus follows:

$$n = 1 + \frac{l_1}{l}\left(1 \pm \sqrt{1 + \frac{l}{l_1}}\right);$$

and combining this equation with (68):

$$\frac{b}{a} = \frac{l_0}{l}\left(1 \pm \sqrt{1 \pm \frac{l}{l_1}}\right). \qquad (69)$$

This equation shows that *Allan's* valve-gear is only correct when the suspension-rods are carried by a lever whose arms have the ratio given by the last formula; it is only then that the variable quantity in equation (64) disappears, and that the centre of oscillation of the valve remains the same for all grades of expansion. The ratio depends only on the length of the eccentric-rod l, the length of the radius-rod l_1, and the position of the point T upon the latter.

For the further investigations, we have to suppose the given condition to be fulfilled by taking the positive sign before the radical as the correct one.

But according to equation (64) the value of O X then becomes

$$\mathrm{OX} = l + l_1 + l_2 - \frac{c^2}{2\,l} - \frac{r^2 \cos^2 \delta}{2\,l}, \qquad (70)$$

and the movement of the valve is therefore

$$\xi = \mathrm{OB} - \mathrm{OX}.$$

Using the value of O B given by equation (63), and putting $u_1 = \frac{u}{n}$, we get, after reduction,

$$\text{(IV}^{\text{a}}\text{)} \qquad \xi = r\left(\sin\delta + \frac{n\,c^2 - u^2}{n\,c\,l}\cos\delta\right)\cos\omega + \frac{u\,r}{c}\left(\cos\delta - \frac{c\,(n-1)}{n\,l}\sin\delta\right)\sin\omega + \frac{r^2}{2\,l}\left(\cos 2\,\delta\sin\omega + \frac{u}{c}\sin 2\,\delta\cos\omega\right)\sin\omega.$$

All quantities containing c have to change sign for *crossed* eccentric-rods.

The third term of the last equation is again the "missing quantity," and it is exactly equal to that which was found for *Stephenson's* valve-gear, and for *Gooch's*. Again, neglecting this quantity, we get for open and crossed rods:

$$\text{(IV)} \qquad \xi = r\left(\sin\delta \pm \frac{n\,c^2 - u^2}{n\,c\,l}\cos\delta\right)\cos\omega \pm \frac{u\,r}{c}\left[\cos\delta \mp \frac{c\,(n-1)}{n\,l}\sin\delta\right]\sin\omega.$$

But in this equation we must have

$$n = 1 + \frac{l_1}{l_0}\,\frac{b}{a}$$

and

$$\text{(IV}^{\text{b}}\text{)} \qquad \frac{b}{a} = \frac{l_0}{l}\left(1 + \sqrt{1 + \frac{l}{l_1}}\right).$$

If we again designate the coefficients of $\cos\omega$ and $\sin\omega$ by A and B, we get as for the other valve-gears:

$$\xi = \mathrm{A}\cos\omega \pm \mathrm{B}\sin\omega. \qquad (71)$$

(b.) *The Curve of Centres.*

The curve in which the centres of all the valve circles belonging to the different grades of expansion are situated may be obtained if the two equations for the co-ordinates

$$a = \frac{\mathrm{A}}{2} = \frac{r}{2}\left(\sin\delta \pm \frac{n\,c^2 - u^2}{n\,c\,l}\cos\delta\right)$$

$$b = \frac{\mathrm{B}}{2} = \frac{r\,u}{2\,c}\left[\cos\delta \mp \frac{c\,(n-1)}{n\,l}\sin\delta\right]$$

are combined by eliminating r. We find that the curve, as in the case of Stephenson's valve-gear, is a *parabola* whose vertex for *open* rods is distant from O by the amount

$$\frac{r}{2}\left(\sin\delta + \frac{c}{l}\cos\delta\right), \tag{72}$$

$y = \frac{nu^2}{2cl}\cos\delta$

and for *crossed* rods by the amount

$$\frac{r}{2}\left(\sin\delta - \frac{c}{l}\cos\delta\right). \tag{73}$$

These formulas are the same as those for Stephenson's gear.

With *open* rods the concave side of the parabola is turned towards O and its parameter is

$$\frac{n\,l\,r}{2\,c\cos\delta}\left(\cos\delta - \frac{c\,(n-1)}{n\,l}\sin\delta\right)^2. \tag{74}$$

With *crossed* rods, the convex side of the parabola is turned towards O, and the parameter is

$$\frac{n\,l\,r}{2\,c\cos\delta}\left(\cos\delta + \frac{c\,(n-1)}{n\,l}\sin\delta\right)^2. \tag{75}$$

Therefore the parabolas are not equal for open and crossed rods as in the case of *Stephenson's* gear. For the method of finding the above formulas, see p. 75.

(c.) *The Suspension of the Link and of the Radius-rod.*

The preceding investigation has shown, that if the construction is to be correct, the two arms S L and S_1 L of the double lever S S_1 must have a certain ratio which is prescribed by the other dimensions of the valve-gear; thus it was shown that

$$\frac{b}{a} = \frac{l_0}{l}\left(1 + \sqrt{1 + \frac{l}{l_1}}\right).$$

The fixed shaft L is placed between the two points of suspension; but *Reuleaux* has shown (Civil-ingenieur, vol. iii.) that the points of suspension S and S_1 can be placed upon *one* side of the turning point; but he adds quite correctly, that this suspension would be impracticable, and we shall not therefore examine it here, but only

discuss the arrangement given by Allan, and assumed in the above calculations.

If we examine, as in the case of the valve-gears already considered, in what curve the two points of suspension S and S_1 must be moved if the vertical movements of the link and sliding-block are to be reduced as much as possible, we arrive at conditions which are not fulfilled by *Allan's* method of suspension, for the two points of suspension should be moved in parabolas or approximately in arcs whose radii are equal to l and l_1, and which present their *convex* sides towards the axle L. This condition could easily be fulfilled practically, by moving the two points S and S_1 in corresponding guides. With regard to the present method of suspension, therefore, there is nothing to recommend, from the theoretical standpoint, but to make the lifting-links as long as possible, and to make the whole length $S S_1$ of the double lever a little greater than the length T K, say,

$$a + b = l_1 - l_0. \tag{76}$$

As the ratio $\frac{b}{a}$ is given by equation (IV^b), the actual lengths of a and b are easily determined.

If the dead point J of the link lies on the direction of the stroke, the lever $S S_1$ is horizontal, and we find for this case, according to equation (32), that the abscissa of the point of suspension S_1, measured from O, must be

$$l - \frac{c^2}{2l}.$$

As the point L in this position lies as much as a farther towards the right, the abscissa of the axis L is, exactly enough for practical purposes,

$$x = l + a - \frac{c^2}{2l}, \tag{77}$$

the length of the two lifting-links giving the corresponding ordinate.

Practical Application of the Diagram.

In order to examine an existing valve-gear, or to design a new one, it is only necessary to know the principal results of the theoretical investigation above given, which results are briefly expressed

by the three following equations. The movement of the valve for any angular movement ω and any grade of expansion u for open or crossed rods, is

$$\text{(IV)} \quad \xi = r\left(\sin\delta \pm \frac{n\,c^2 - u^2}{n\,c\,l}\cos\delta\right)\cos\omega$$
$$\pm \frac{u\,r}{c}\left(\cos\delta \mp \frac{c\,(n-1)}{n\,l}\sin\delta\right)\sin\omega,$$

when the different symbols, with the exception of u, have the signification above given (p. 124).

But it is necessary that

$$\text{(IV}^{\text{b}}\text{)} \qquad n = 1 + \frac{l_1}{l_0}\frac{b}{a},$$

and that

$$\frac{b}{a} = \frac{l_0}{l}\left(1 + \sqrt{1 + \frac{l_1}{l}}\right).$$

Now, in order to ascertain by the aid of the diagram the distribution of the steam for each grade of expansion, we must again draw the different valve circles whose centres have the co-ordinates

$$\text{O B} = a = \frac{r}{2}\left(\sin\delta \pm \frac{n\,c^2 - u^2}{n\,c\,l}\cos\delta\right),$$

$$\text{B C} = b = \frac{r\,u}{2\,c}\left[\cos\delta \mp \frac{c\,(n-1)}{n\,l}\sin\delta\right].$$

An example will clearly show the application of this formula.

Problem.—An existing valve-gear, *Allan's* system, has the following dimensions (Fig. 13, Pl. III.):—

Eccentricity	$r = 0{\cdot}070^{\text{m}}$ (2·75″)
Angle of advance	$\delta = 30^\circ$
Length of the eccentric-rods	$l = 1{\cdot}250^{\text{m}}$ (49″)
Half-length of the link .. C J = C J_1 =	$c = 0{\cdot}150^{\text{m}}$ (6″)
Length of the radius-rod	$l_1 = 1{\cdot}500^{\text{m}}$ (60″)
Distance of the point T from the end B_1 of the radius-rod	$l_0 = 1{\cdot}250^{\text{m}}$ (49″)
Lifting-arm S_1 L = a of the lever S S_1 ..	$a = 0{\cdot}075^{\text{m}}$ (2·9″)
Lifting-arm S L = b	$b = 0{\cdot}175^{\text{m}}$ (6·8″)
Outside lap	$e = 0{\cdot}024^{\text{m}}$ (0·9″)
Inside lap	$i = 0{\cdot}005^{\text{m}}$ (0·19″)

The valve-gear has crossed eccentric-rods, and four grades for the forward gear and as many for the back gear.

The sliding-block K stands at C or C_1 when the motion is in full gear; for these positions $u = +c$ or $-c$.

The distribution of the steam for all grades of expansion is to be examined, and in addition any peculiarities this valve-gear may possess are to be ascertained from the diagram.

We must first examine whether equation (IV_b) is fulfilled, for then only is the valve-gear useful; if it were not fulfilled, the valve would not, at the different grades of expansion, move about the same fixed point. We must therefore have

$$\frac{b}{a} = \frac{l_0}{l}\left(1 + \sqrt{1 + \frac{l}{l_1}}\right).$$

Substituting the given values, we have

$$\frac{b}{a} = 2 \cdot 35.$$

The given values of a and b have the same ratio, and on this score, therefore, no fault can be found with the valve-gear.

If we now calculate

$$n = 1 + \frac{l_1}{l_0}\frac{b}{a},$$

we get

$$n = 2 \cdot 82.$$

For calculating the co-ordinates of the centres of the valve circles, the following formulas must be used, for the eccentric-rods are crossed:

$$\mathrm{OB} = a = \frac{r}{2}\left(\sin\delta - \frac{n c^2 - u^2}{n c l}\cos\delta\right)$$

$$\mathrm{BC} = b = \frac{r u}{2 c}\left(\cos\delta + \frac{c(n-1)}{n l}\sin\delta\right).$$

From these formulas we can now get the co-ordinates for the 4th or last grade of expansion, thus for $u = c$ and the other known values, we have

$$\mathrm{OB} = 0 \cdot 0151^{\mathrm{m}}\ (0 \cdot 60'') \quad \text{and} \quad \mathrm{BC} = 0 \cdot 0316^{\mathrm{m}}\ (1 \cdot 24'').$$

Substituting in the corresponding formulas, successively,

$$u = \tfrac{3}{4}c, \quad u = \tfrac{2}{4}c, \quad u = \tfrac{1}{4}c, \quad \text{and } u = 0,$$

we obtain the co-ordinates of the centres of the valve circles

For the 3rd grade	$O B_3 = 0 \cdot 0146^m$ $(0 \cdot 51'')$	$B_3 C_3 = 0 \cdot 0237^m$ $(0 \cdot 9'')$
„ 2nd „	$O B_2 = 0 \cdot 0138^m$ $(0 \cdot 54'')$	$B_2 C_2 = 0 \cdot 0158^m$ $(0 \cdot 6'')$
„ 1st „	$O B_1 = 0 \cdot 0135^m$ $(0 \cdot 53'')$	$B_1 C_1 = 0 \cdot 0079^m$ $(0 \cdot 3'')$
„ dead point	$O B_0 = 0 \cdot 0132^m$ $(0 \cdot 52'')$	$0 \cdot$

The co-ordinates are now laid off as in Fig. 14, Pl. III., and the centres C_0, C_1, C_2, C_3, and C of the valve circles are thus obtained. Describe next the lap circles with the laps $O V = e = 0 \cdot 024^m$ $(0 \cdot 94'')$ and $O W = i = 0 \cdot 005^m$ $(0 \cdot 23'')$ as radii, and the diagram is complete. In order to ascertain from the diagram the distribution of the steam, we refer the reader to the explanations accompanying the diagrams (Figs. 8 and 10, Pl. II.), for *Stephenson's* valve-gear, and to the remarks on p. 89 and those following.

But we may call attention to one peculiarity of *Allan's* valve-gear with regard to the lead; it will be seen that in the present case, *crossed* rods, the lead *decreases* the higher the expansion; thus, while the lead for the 4th grade is $0 \cdot 006^m$ $(0 \cdot 27'')$, for the 1st grade it is only $0 \cdot 003^m$ $(0 \cdot 11'')$; just the opposite result would obtain for open rods. It will thus be seen that *Allan's* valve-gear is in this respect similar to *Stephenson's;* but the variation of the lead, other things being equal, may be easily shown to be always less in *Allan's* than in *Stephenson's* gear.

But this is not sufficient reason for considering Allan's valve-gear the better one, for the small advantage with respect to the lead is rather dearly purchased, as in this, as well as in Gooch's valve-gear, circumstances prevent its application in many cases, because the insertion of the long radius-rod requires a much greater distance between steam-chest and driving-axle than in Stephenson's valve-gear.

There is no difficulty in ascertaining, according to the method explained above, the different dimensions which are to be chosen in designing a valve-gear. What has already been said about *Gooch's*

valve-gear, is in general correct for *Allan's* gear also; it is only necessary to add, that the ratio $\frac{b}{a}$ of the lever-arms must be so chosen that the equation

$$\frac{b}{a} = \frac{l_0}{l}\left(1 + \sqrt{1 + \frac{l}{l_1}}\right)$$

is satisfied.

The above investigations have shown how similar the constructions of the three valve-gears by *Stephenson*, *Gooch*, and *Allan* are; we found, notwithstanding the different arrangements, for all three valve-gears the same "missing quantity"; the dead point of the link governs, under equal circumstances, the movement of the valve in the same manner in the one as in the other of the valve-gears, and the respective diagrams show such coincidence that we arrive at the conclusion that in respect to the distribution of the steam none of these gears deserves the preference.

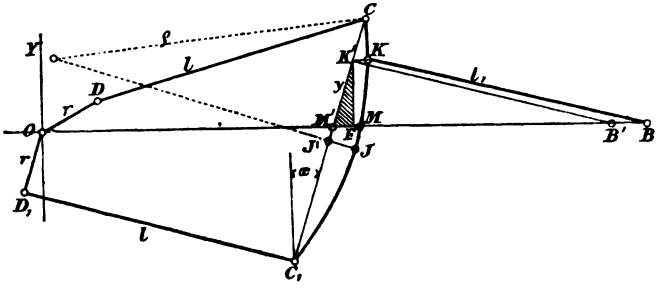
The close relationship existing between these three valve-gears may be easily described. In *Stephenson's* valve-gear the link is raised and lowered while the radius-rod remains in the direction of the stroke; in *Gooch's* valve-gear the link has no vertical motion, it is the radius-rod which is raised and lowered; in *Allan's* valve-gear the two motions take place simultaneously and in such a manner that when the link is raised the radius-rod is lowered and *vice versâ*.

In what has preceded we have discussed the three valve-gears in the order in which they were invented, and have developed for each separately the formulas for the movement of the valve; but it is perfectly easy to so conduct the analytical development that it will include the three cases; the close relationship, then, finds expression in the fundamental formulas, which also show whether the three cases are the only possible ones, or whether it is possible to effect an improvement by altering the general arrangement of the parts.

The analytical treatment just mentioned is particularly suited for collegiate lectures, and the author has employed this method in his lectures for years; it will now be given in order that the problem may be examined from every point of view.

As the basis of the investigation, we will assume *Stephenson's* valve-gear, with open rods; but we will also assume, as in *Gooch's* and *Allan's* gears, that a radius-rod B K (Fig. 35) is inserted between valve-stem and link, which rod, like the link, is raised and lowered.

FIG. 35.



We will now, as in the separate discussions above mentioned, deduce the distance O B for any crank angle ω; as before, let δ represent the angle of advance, r the eccentricity, l the length of the eccentric-rods C D and C_1 D_1, and $2c$ the length of the link C C_1, which may be assumed to be the same as the chord C C_1, provided the link is curved to as great a radius ρ as possible.

Let us further assume that the link is lowered the amount J M = J′ M′ = x, and that the end K of the radius-rod is raised above the direction of the stroke O B the amount M K = M′ K′ = y, then we have from the figure

$$\mathrm{O\,B} = \mathrm{O\,M'} + \mathrm{M'F} + \mathrm{F\,B'} + \mathrm{B'\,B}. \qquad (78)$$

The distance O M′ can be determined as in the investigation of *Stephenson's* valve-gear, p. 71; to avoid repetition in establishing the value O M′, we at once make use of equation (18), but drop its last term because the "missing quantity" has been neglected from the start; as the lowering of the link is here represented by x instead of u, we substitute in this equation x in place of u, also make $\rho = \infty$, because M′ lies on the chord of the link, and because we may at first regard the link as straight; then we have

$$\mathrm{O\,M'} = r\left(\sin\delta + \frac{c^2 - x^2}{c\,l}\cos\delta\right)\cos\omega$$
$$+ \frac{x\,r}{c}\cos\delta\sin\omega + l - \frac{c^2 - x^2}{2\,l}. \qquad (79)$$

Moreover the figure gives

$$M' F = y \sin a.$$

where, as before, a represents the inclination of the chord of the link to a perpendicular let fall on the direction of the stroke; employing equation (16), again neglecting its last term, and substituting x for u, we obtain

$$M' F = \frac{y r}{c} \cos \delta \sin \omega - \frac{y r}{l} \sin \delta \sin \omega$$
$$- \frac{x y r}{c l} \cos \delta \cos \omega + \frac{x y}{l}. \qquad (80)$$

If we represent the length B′ K′ (Fig. 35) by l_1, we get

$$F B' = \sqrt{l_1^2 - y^2 \cos^2 a}.$$

But as a is always very small, we can assume $\cos a = 1$, and as l_1 is very large in comparison with the lift y of the radius-rod, we may develop the radical according to the binomial theorem and employ only the first two terms of the series, we then have

$$F B' = l_1 - \frac{y^2}{2 l_1}. \qquad (81)$$

Finally, we must take into account that the link is not straight, but is curved to an arc having the radius ρ, and that, consequently, one end of the radius-rod must be shifted from K′ to K and the other end from B′ to B, so that B′ B = K′ K. The distance K′ K can be determined approximately as follows:—The half-length c of the chord corresponds to the rise $J J' = \frac{c^2}{2 \rho}$ of the arc of the link, on the other hand the half chord $J' K' = x + y$ corresponds to the rise

$$\frac{(x + y)^2}{2 \rho};$$

the difference between these rises, it is easy to see, represents the distance K′ K, hence

$$B' B = K' K = \frac{c^2}{2 \rho} - \frac{(x + y)^2}{2 \rho}. \qquad (82)$$

Now, combining equations (79) to (82) with (78), we get after reduction

$$\mathrm{OB} = r\left(\sin\delta + \frac{c^2 - x^2 - xy}{cl}\cos\delta\right)\cos\omega + r\left(\frac{x+y}{c}\cos\delta - \frac{y}{l}\sin\delta\right)\sin\omega$$
$$+ \left(l + l_1 - \frac{c^2}{2l} + \frac{c^2}{2\rho} + \frac{x^2 + 2xy}{2l} - \frac{y^2}{2l_1} - \frac{(x+y)^2}{2\rho}\right). \qquad (83)$$

Now this equation shows that the valve-gear arrangement sketched in Fig. 35 (though it does not occur in this form in practice) might well be employed under certain conditions. The first two terms represent the well-known law of valve motion

$$\xi = \mathrm{A}\cos\omega + \mathrm{B}\sin\omega,$$

where

$$\left.\begin{aligned} \mathrm{A} &= r\left(\sin\delta + \frac{c^2 - x^2 - xy}{cl}\cos\delta\right) \\ \mathrm{B} &= r\left(\frac{x+y}{c}\cos\delta - \frac{y}{l}\sin\delta\right) \end{aligned}\right\}; \qquad (84)$$

moreover the point B, which could be employed to drive the valve, oscillates about a centre of motion whose distance from the axis O is given by the third term in equation (83).

If, as in previous investigations, we designate this centre of motion by X, its distance from the driving-axle is given by

$$\mathrm{OX} = l + l_1 - \frac{c^2}{2l} + \frac{c^2}{2\rho} + \frac{x^2 + 2xy}{2l} - \frac{y^2}{2l_1} - \frac{(x+y)^2}{2\rho}. \qquad (85)$$

Evidently, the valve-gear can only be employed as a link-motion when the centre of motion X occupies the same position for *every* position of the link and radius-rod, i. e. for every possible value of x and y; or, in other words, it can only be thus employed when the distance OX is constant for all values of x and y. Consequently the only condition to be fulfilled, in order to render the valve-gear sketch in Fig. 35 practically useful, is that the last three terms in equation (85) disappear, which gives the equation of condition

$$\frac{x^2 + 2xy}{2l} - \frac{y^2}{2l_1} - \frac{(x+y)^2}{2\rho} = 0;^*$$

* It would be still more general to place this equation equal to a constant instead of equal to zero; but then results are obtained which would lead to very complicated arrangements for effecting the proper relative motion of link and radius-rod, for the distance x is then no longer proportional to y.

if to the left member of this equation we add and subtract $\frac{y^2}{2l}$, and reduce, we get

$$(x+y)^2\left(1-\frac{l}{\rho}\right)-y^2\left(1+\frac{l}{l_1}\right)=0, \qquad (86)$$

and this condition is always easily fulfilled.

We will now examine a few special cases.

Case I.—The radius-rod B K (Fig. 35) is neither raised nor lowered, its end K being constantly kept in the direction of the stroke, then $y = 0$, and the equation of condition (86) becomes

$$x^2\left(1-\frac{l}{\rho}\right)=0,$$

and this equation can only be satisfied when ρ is made equal to l, i. e. when the link is curved to an arc whose radius is equal to the length of the eccentric-rod, consequently *this case corresponds to Stephenson's link-motion.* If we substitute in equations (84) $y = 0$ and x for u, we obtain the formulas (22) and (23), p. 73, which were deduced above for the valve movement in *Stephenson's* link-motion.

Case II.—The radius-rod is raised and lowered, but the dead point of the link is kept constantly on the line of the stroke. For this case we must substitute $x = 0$ in the above general equations, and equation of condition (86) gives

$$\left(1-\frac{l}{\rho}\right)-\left(1+\frac{l}{l_1}\right)=0,$$

or

$$\rho=-l_1.$$

The link must therefore be curved according to an arc whose radius is equal to the length of the radius-rod, and the negative sign shows that the curvature is in the opposite direction, i. e. its convex side must be turned toward the driving-axle. This case corresponds to *Gooch's link-motion.* Substituting $x = 0$ and $y = u$ in equation (84) we get the formulas (45) and (46), which were deduced for *Gooch's* link-motion.

Case III.—The link and radius-rod both move vertically at the

same time (as was assumed in developing the general equation), but the link is straight; then $\rho = \infty$, and equation (86) gives for the equation of condition

$$\frac{x+y}{y} = \sqrt{1 + \frac{l}{l_1}},$$

or

$$\frac{x}{y} = -1 + \sqrt{1 + \frac{l}{l_1}};$$

i. e. the ratio of the distance through which the link is lowered to the distance through which the radius-rod is raised, must be constant and equal to the ratio prescribed by the last formula. *This corresponds to Allan's link-motion.*

If we substitute $x + y = u$, also $u = n\,x$ where n represents a constant quantity, we obtain from the preceding equations and the equations (84) the formula IV^a (p. 128), which was before directly deduced for *Allan's* valve-gear.

General Case.—A closer examination of the equation of condition (86), shows, however, that the arrangements invented by *Stephenson, Gooch,* and *Allan,* are not the only possible ones, but that there are an *infinite number of cases* varying with the choice of the radius ρ of the link.

If we wish to actually construct the arrangement sketched in Fig. 35, it is only necessary that equation (86) should be satisfied, which equation may be also written in the following general manner:

$$\frac{x+y}{y} = \sqrt{\frac{1 + \frac{l}{l_1}}{1 - \frac{l}{\rho}}}.$$

It at once follows from this equation that y must always be directly proportional to x; the ratio of the two values is constant and can be easily obtained from the last formula. Moreover, it is evident that the radius ρ of the arc of the link can be assumed equal or greater than the length of the eccentric-rods when, as assumed in the development, the link turns its *concave* side to the driving-axle, for $\rho < l$ would make the radical imaginary. On the other hand, when

the link is turned about, i. e. when its *convex* side is turned towards the axle, we have

$$\frac{x+y}{y} = \sqrt{\frac{1+\frac{l}{l_1}}{1+\frac{l}{\rho}}}.$$

If, as will always be the case in practice, we suppose the link to be lowered when the radius-rod is raised, or *vice versâ*, the preceding radical becomes greater than unity, and then ρ must be greater than the length l_1 of the radius-rod, if the limiting case ($\rho = l_1$) represented by *Gooch's* gear is to be avoided.

This will suffice for the general case, for it is evident that the preceding formulas are applicable to crossed rods (when an opposite sign is given to the values of c in the formulas), and that the value of x becomes negative when link and radius-rod are simultaneously raised or lowered; the object of this investigation is only to show that the formulas for *Stephenson's*, *Gooch's*, and *Allan's* link-motions are but special cases of more general formulas, and to point out the interesting fact that among the infinite number of possible cases practice has picked out by trial just those *three* cases which have been found to be theoretically the simplest and most feasible.

We said above that, as regards the distribution of steam, neither *Stephenson's*, *Gooch's*, nor *Allan's* link-motion deserved the preference. In respect to this distribution *one* doubt may still arise, and that is with reference to the lead; the question is often put, whether a constant lead for all grades is really of such importance for an advantageous distribution of the steam as is believed by many, and whether, if a constant lead is not possible, a greater or smaller one at a higher expansion is to be permitted, i. e. whether in the case of *Stephenson's* and *Allan's* valve-gears, open or crossed eccentric-rods should be applied. A correct answer to this question can perhaps only be obtained through exact experiments; for the present a certain amount of the lead must be taken as advantageous, but it must also be admitted, according to the calculations by *Reuleaux* ('Civil-ingenieur,' vol. iii.), that the slight variations given by reversing-motions with variable expansion, have no disadvantageous influence upon the

action of the steam in the cylinder. Therefore, *Stephenson's* and *Allan's* valve-gears are not in this respect inferior to *Gooch's*, supposing the dimensions are so chosen that the lead does not increase too much, or disappear entirely, as may happen when the outside lap with crossed rods is not correctly determined.

In conclusion, we must add a few words about a peculiarity of a few existing locomotive valve-gears; namely, the laps are sometimes *different* on the two sides. Such an arrangement may be a necessity when the principal positions of the crank, in consequence of a *very short connecting-rod*, differ too much during the forward and return strokes, and when the beginning of the expansion and of the exhaust for each stroke are to take place at *the same piston positions*, i. e. *when the piston is at the same distance from the beginning of the stroke.*

But if the connecting-rod is long enough, there is no reason for the above arrangement, unless a fault in the design of the valve-gear should require such an alteration. But this fault is probably to be found in an incorrect suspension of the link or of the radius-rod supposing that we have *Stephenson's* or *Gooch's* system, but if we have *Allan's* valve-gear, the ratio of the arms of the double lever which carries the suspension-rods may not have been chosen so as to correspond with the formula given above. If the suspension is a correct one, and if the eccentric-rods are sufficiently long, our theory gives, even if the "missing quantity" is taken into account, no results which indicate the necessity of such an arrangement. If the valve is adjusted to equal lead for *one* grade, then it is equal on both sides of the valve for all other grades, even when the eccentric-rods are short, as has been proved by theory and experiments. Of course it is another question, whether with short eccentric-rods the distribution of the steam will be an advantageous one for *all* grades of expansion; but this question is most decidedly to be answered in the negative. No possible alteration in the valve can in the least improve the distribution of the steam in such valve-gears.

In the following pages we give the investigation of two link-motions, whose parts are quite differently arranged from those already examined.

CHAPTER IV.

Heusinger von Waldegg's Link-motion.

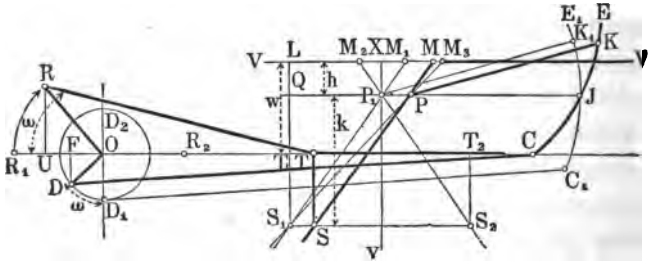
Description of the Valve-gear.

This valve-gear is represented in side elevation by Fig. 15, Pl. III. O is the centre of the driving-axle, upon which is fixed the crank O R, and an eccentric E whose eccentricity O D forms a right angle with the centre line of the crank.

The eccentric-rod D C gives to the link C C, which turns on the fixed pin J, an oscillating motion. The link has a curved slot in which the sliding-block K can be moved up and down by means of the lifting-link K H, so as to produce a variable expansion; the block is jointed to the radius-rod P K.

The radius-rod P K is connected at its end P with two levers M S, whose upper ends are jointed to the valve-spindle V V, while their lower ends turn in bearings S on the cross-head W. Each lever M S has thus a very peculiar motion given to it; the end S moves back and forth with the piston-rod, the point P gets from the link another oscillating motion, and the second point M is thus given a motion which, as experience has shown, is suitable for working the valve.

Fig. 36.



The different parts are represented in Fig. 36 by lines only, but are distinguished by the same letters when in different positions. If the crank O R stands at one of the dead points, thus in the position O R_1, the piston is at the beginning of the stroke, and the arm of the

cross-head is at $T_1 S_1$. The centre line of the eccentric is at that moment at $O D_1$, the link occupies the central position $C_1 J E_1$, the radius-rod is in the position $K_1 P_1$, and the lever which governs the valve-stem is at $S_1 P_1 M_1$. If the crank is now turned through 180°, i. e. into the position $O R_2$, the cross-head will have arrived at the other end of the stroke or at $T_2 S_2$, the link will again occupy its central position because the centre line of the eccentric has arrived at $O D_2$. The radius-rod will again occupy the position $K_1 P_1$, and the lever which governs the valve will be at the position $S_2 M_2$, i. e. the valve will have travelled the distance $M_1 M_2$ during the supposed movement from right to left. If we bisect $M_1 M_2$ at X, this point marks the central position of the valve, for the movement of the valve must be a symmetrical one if we imagine the centre of the valve to be directly fastened to the end M of the motion-lever M S. Moreover, the straight line $X P_1 Y$ lies at the centre of the piston stroke $S_1 S_2$. The distances $M_1 X$ and $M_2 X$, therefore, represent the movements of the valve from its central position, *when the crank stands at the dead points.* As each of the two distances $M_1 X$ and $M_2 X$ represents the lead *plus* the outside lap, and as the first has to be constant for all grades of expansion, the position of the lever $M_1 S_1$ or $M_2 S_2$ has to remain unaltered when the end K_1 of the radius-rod $K_1 P_1$ is moved up and down in the link while the crank stands at one of the dead points; the point P_1 must not, therefore, alter its position while this raising or lowering of the radius-rod takes place. It will be at once seen that this is the case when the slot in the link is curved to an arc whose radius is equal to the length $K_1 P_1$ of the radius-rod. Moreover, it is best to put the turning-point J of the link on a line $J P_1$ parallel to the direction of the valve-stem V V.

If we follow by calculation the motion of the valve during any angular movement of the crank, it will be found that establishing formulas for the movement of the valve is connected with great difficulties, but if we are willing to approximate, the relations sought become very simple.

Let the eccentricity $O D = r$ (Fig. 36), the radius of the crank $O R = R$; the half-length J C of the link $= c$, the distance J K of the sliding-block K from the centre of the link for any grade of

expansion $= u$. Next let $P_1 X = h$ be the distance of the point P_1 from the valve-stem V V at the central position of the link, and $P_1 Y = Q S_1 = k$ the distance of the centre of the bearing S from the line J Q which is parallel to the valve-stem. Finally, let us assume an eccentric and radius-rod as long as possible.

Theory of Heusinger von Waldegg's Link-motion.

(a.) *Determination of the Movement of the Valve.*

If we start from the position $O R_1$ of the crank, the link at that moment occupies the position $C_1 E_1$, the motion-lever is in the position $S_1 M_1$ and the centre of the valve is at the distance $M_1 X$ from its central position. Now, instead of supposing the crank and the eccentric to travel together through an angle ω, we shall imagine, in order that the matter may be better understood, that they turn separately. If we suppose the crank for the present to be fixed at $O R_1$ and turn the eccentricity r in the direction of the arrow through the angle $D_1 O D = \omega$, the point D of the eccentric-rod, and therefore its point of connection C with the link, will move, approximately, as much as the distance O F to the left, and this distance is $= r \sin \omega$. While the point C moves as much as $C C_1 = r \sin \omega$ to the left, the link arrives at the position C J E, and the sliding-block K_1 moves to K, so that

$$\frac{K K_1}{J K} = \frac{C C_1}{J C},$$

or according to our notation $K K_1 = \frac{u}{c} r \sin \omega$. The movement of the point K_1 causes the other end P_1 of the radius-rod to move to P, so that approximately $P P_1 = K K_1 = \frac{u}{c} r \sin \omega$. Now, as the crank is for the present supposed to be fixed at $O R_1$, the arm $S_1 T_1$ or bearing S_1 will not alter its position, and the motion-lever will pass, in consequence of the movement of P_1, from the position $S_1 P_1 M_1$ to $S_1 P M_3$ and the centre of the valve from M_1 to M_3, provided the crank has not at the same time turned through the angle $R O R_1$. Let us first determine the movement $M_1 M_3$ of the valve. During

the movement of P_1 towards the right, the point P_1 leaves the straight line $P_1 J$, for $M_1 P_1 = M_3 P$, but as this deviation is always very small, it may be assumed approximately that the point P_1 always moves like M_1 back and forth in a straight line, and therefore that $P_1 P$ is parallel to $M_1 M_2$. Then follows, according to Fig. 36,

$$\triangle\ M_1 S_1 M_3 \text{ similar to } \triangle\ P_1 S_1 P,$$

hence

$$\frac{M_1 M_3}{P P_1} = \frac{M_1 S_1}{P_1 S_1} = \frac{X Y}{P_1 Y},$$

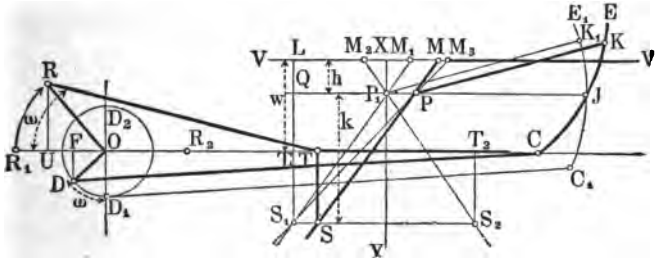
or, according to the notation, and because

$$P P_1 = \frac{u}{c} r \sin \omega,$$

we have

$$M_1 M_3 = \frac{h + k}{k} \cdot \frac{u}{c} r \sin \omega. \tag{87}$$

FIG. 36.



But M_3 is not the true position of the centre of the valve; for if the crank is now turned through the angle $R_1 O R = \omega$, the arm $S_1 T_1$ advances to $S T$, the motion-lever moves in the bearing S, while it (the motion-lever) turns round the point P, which is at present supposed to be fixed, and brings the centre of the valve from M_3 to M, i. e. to the place it would have reached if the movements of the crank and eccentric had taken place simultaneously. The distance $M M_3$ may now be easily calculated. We have

$$\triangle\ M M_3 P \text{ similar to } \triangle\ S P S_1,$$

therefore

$$M M_3 : S S_1 = M_3 P : P S_1,$$

or, because

$$M_3 P : P S_1 = X P_1 : P_1 Y = h : k,$$

$$M M_3 = \frac{h}{k} . S S_1.$$

But $S S_1$ is simply the movement of the piston for the angular movement ω, hence

$$S S_1 = T T_1 = U R_1 = R (1 - \cos \omega) \text{ approximately.}$$

The influence of the length of the connecting-rod may here be neglected, as the quantity $S S_1$ has to be multiplied by the small fraction $\frac{h}{k}$ in order to determine $M M_3$. If we substitute the value of $S S_1$ in the above equation, we get for the latter

$$M M_3 = \frac{h}{k} R (1 - \cos \omega). \tag{88}$$

The figure shows that at the present position O R of the crank, the valve is at the distance M X from the central position, and we therefore have

$$M X = M_1 X + M_1 M_3 - M M_3. \tag{89}$$

But we also have

$$\frac{M_1 X}{X P_1} = \frac{Y S_1}{Y P_1},$$

or, because

$$Y S_1 = R \quad \text{and} \quad Y P_1 = k,$$

$$M_1 X = \frac{h}{k} R.$$

If we substitute this value, as well as the results of equation (87) and (88), in equation (89) and reduce, we get for the movement $M X = \xi$ of the valve from the centre of its stroke during the angular movement ω

$$\text{(V)} \qquad \xi = \frac{h}{k} R \cos \omega + \frac{u}{c} \frac{h + k}{k} r \sin \omega.$$

If we put for simplicity

$$\frac{h}{k} R = A, \tag{90}$$

and for a certain grade of expansion, i. e. a given value of u,

$$\frac{u}{c}\,\frac{h+k}{k}\,r = \mathrm{B}, \tag{91}$$

the equation for the movement of the valve will be

$$\xi = \mathrm{A}\cos\omega + \mathrm{B}\sin\omega. \tag{92}$$

(b.) *The Curve of Centres.*

The abscissa for any point of the curve of centres, or, in other words, for the centre of any valve circle, is, according to a preceding proposition,

$$\mathrm{O\,B} = a = \frac{1}{2}\,\mathrm{A} = \frac{1}{2}\,\frac{h}{k}\,\mathrm{R};$$

and the ordinate

$$\mathrm{B\,C} = b = \frac{1}{2}\,\mathrm{B} = \frac{u}{2\,c}\,\frac{h+k}{k}\,r,$$

for the formula for the movement of the valve of the present valve-gear has also the general form

$$\xi = \mathrm{A}\cos\omega + \mathrm{B}\sin\omega,$$

the movements of the valve are, therefore, represented here also by chords of circles, which we have called valve circles.

As the formula for the abscissa does not contain the value of u, all points of the curve of centres have the same abscissa, and it follows that the required "curve" of the present valve-gear, as in *Gooch's, is a straight line* $B\,C_4$ (Fig. 33), *which is perpendicular to the line of motion* $O\,X$.

Practical Application of the Diagram.

In practice it is only necessary, in order to be able to examine an existing valve-gear of the present system, or to design a new one, to know the equation for the movement of the valve.

The equation is

$$\xi = \frac{h}{k}\,\mathrm{R}\cos\omega + \frac{u}{c}\,\frac{h+k}{k}\,r\sin\omega.$$

The different letters have the significations already given, and only with respect to u is it necessary to add a few words, for the link may be so constructed, that at the last grade, i. e. when the radius-rod is completely raised or lowered, the slide-block K is either less remote from the dead point than the point of connection C of the eccentric-rod, or this distance is greater than C J. The first case occurs in the valve-gears constructed by *Heusinger*. We shall, as before, designate by c the distance of the slide-block from the dead point at the last grade, and at once explain the remaining features of this valve-gear by a particular example.

We have chosen for this purpose a valve-gear employed by Heusinger von Waldegg on one of his small tank-engines. The principal dimensions are—

Crank radius	$R = 0 \cdot 14^m$ (5·5″)
Eccentricity	$r = 0 \cdot 032^m$ (1·28″)
Half-length of the link .. .. J C =	$c = 0 \cdot 108^m$ (4·32″)
Distance of the last grade of expansion from the dead point of the link, thus .. .. J K =	$c_1 = 0 \cdot 081^m$ (3·18″)
Vertical distance of the point P from the axis of the valve-stem	$h = 0 \cdot 028^m$ (1·16″)
Vertical distance of the point P from the centre of the bearing S	$k = 0 \cdot 304^m$ (12″)
Outside lap	$e = 0 \cdot 011^m$ (0·43″)
Inside lap	$i = 0 \cdot 002^m$ (0·07″)

If we assume four grades of expansion for the forward gear and four for the back gear, i. e. $n = 4$, all questions with respect to the distribution of the steam may be easily answered from these data.

We must first calculate the co-ordinates for the centre of the valve circle of the last or fourth grade by substituting $u = c$ in the following formulas:

$$BO = \frac{1}{2}\frac{h}{k}R$$

and

$$BC = \frac{1}{2}\frac{u}{c}\frac{h+k}{k}r.$$

Substituting the data given above, we get

$$O\,B = 0{\cdot}00645^{m}\ (0{\cdot}25'')$$

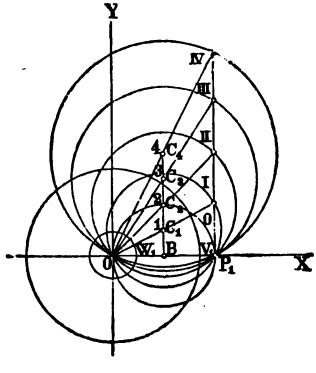
and

$$B\,C_4 = 0{\cdot}0131^{m}\ (0{\cdot}52''),$$

and laying off these values in Fig. 37 we get the centre C_4 of the valve circle for the last grade.

This constitutes the whole of the calculation, for the centres for the valve circles of the other grades are obtained simply by dividing the ordinate B C_4 into as many parts as there are grades, in this case into *four*. If we now describe from C_4, C_3, C_2, C_1, and B circles with C_4 O, C_3 O, &c., &c., as radii, and also from O the lap circles with the laps *e* and *i* as radii, the diagram is complete, and we may proceed to investigate the distribution of the steam in exactly the same manner as in the case of the valve-gears already described.

FIG. 37.



The diagram is exactly the same as that for *Gooch's* valve-gear. As all valve circles cut the axis of the abscissa at the same point P_1, the lead $P_1\,V_1$ is constant for all grades, and that was just what *Heusinger* sought to accomplish. In the valve-gear under examination the outside lead is equal to $0{\cdot}002^{m}$ $(0{\cdot}078'')$, and the inside one $0{\cdot}011^{m}$ $(0{\cdot}43'')$.

The method of ascertaining the distribution of the steam from the above-given dimensions of the valve-gear has been already sufficiently explained.

But it is of special importance to determine for certain conditions certain parts of the valve-gear, for example the outside and inside lap, the eccentricity, &c. We can safely assume that in practice in the formula for the movement of the valve

$$\xi = \frac{h}{k}\,R\cos\omega + \frac{u}{c}\,\frac{h+k}{k}\,r\sin\omega,$$

the length R of the crank is known and also the values of h and k (Fig. 36). It will only be necessary to take care that h is as small and k as large as possible. Next the vertical distance $L T_1 = w$ between the centre lines of the valve-stem and piston-rod is to be taken as given, and also the half-length of the link, which with sufficient accuracy may be assumed as $c = C_1 J = T_1 Q = w - h$. Moreover, any value may be chosen for c_1, i. e. for the distance of the last grade from the dead point J of the link.

The method which has to be followed in designing a new valve-gear is indicated by the following example.

In a valve-gear on *Heusinger von Waldegg's* system it is required that steam shall be admitted at the last grade of expansion while the piston travels through 0·825 of its stroke, and that the exhaust shall begin when the piston has still to travel through 0·045 of its stroke. There is given

$$\begin{aligned} R &= 0{\cdot}14^m\ (5{\cdot}5'') \\ c &= 0{\cdot}108^m\ (4{\cdot}32'') \\ c_1 &= 0{\cdot}081^m\ (3{\cdot}18'') \\ h &= 0{\cdot}028^m\ (1{\cdot}1'') \\ k &= 0{\cdot}304^m\ (12'') \\ \text{the outside lead} &= 0{\cdot}002^m\ (0{\cdot}078''). \end{aligned}$$

The eccentricity r, as well as the outside and inside lap, are to be determined, and then the distribution of the steam for all other grades of expansion is to be examined, there being four grades for the forward gear and four for the back gear.

Draw the two axes O M and $O P_2$ (Fig. 38), and describe, to any scale, the crank-pin circle $M R R_1$. If we suppose the piston to travel on the diameter from M to N, the admission of the steam must cease at M_1, so that $M M_1 = 0{\cdot}825 \,.\, M N$. Erect the perpendicular $M_1 R_1$, meeting the crank-pin circle at R_1, and connect R_1 with O by the straight line $O R_1$; this is then the position of the crank at the beginning of the expansion. Calculate next

$$A = \frac{h}{k} R = 0{\cdot}0129 = O P_1.$$

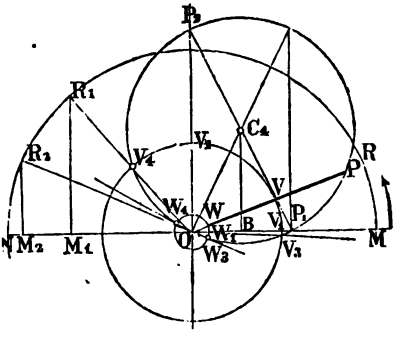
If we lay off the value of the outside lead $V_1 P_1 = 0{\cdot}002^m$ (0·07″) from P_1 towards O, then $O V_1 = e$ is at once the required outside

lap, which is found by measurement to be $e = 0 \cdot 011^m$ ($0 \cdot 43''$) if we assume that Fig. 38 is drawn full-size.

Describe from O, with the lap e as a radius, the circle $V_1 V_4$; it intersects the position $O R_1$ of the crank at V_4; now drawing a circle through the three points V_4, O and P_1 we get the valve circle corresponding to the 4th grade. For the distance $B C_4$ of its centre from the axis O M we have given

$$b = \frac{1}{2} B = \frac{1}{2} \frac{c_1}{c} \frac{h + k}{k} r.$$

Fig. 38.



The distance $b = B C_4$ is to be measured; for the present case $b = 0 \cdot 013^m$ ($0 \cdot 57''$), therefore from the last equation we get

$$r = \frac{c}{c_1} \frac{k}{k + h} 2 b.$$

Substituting the known values in the right-hand member of the equation we obtain the required eccentricity

$$r = 0 \cdot 032^m \ (1 \cdot 25'').$$

In order to determine the *inside* lap i, it is to be observed that the exhaust must *begin* when the piston has still $0 \cdot 045$ of its stroke to travel. Make therefore $N M_2 = 0 \cdot 045$ M N, erect at M_2 the perpendicular $M_2 R_2$, and produce $O R_2$ beyond O to the point of intersection W_3 with the valve circle. $O W_3$ is the required inside lap, which measurement shows to be

$$O W_3 = i = 0 \cdot 002^m \ (0 \cdot 08'').$$

Describing the circle $W_3 W_4$ with i as a radius, and next the other valve circles, it will be easy to at once inspect the distribution of the steam for all grades of expansion.

The examination of the diagram of *Heusinger's* valve-gear shows that the aim, to produce a constant lead, is completely attained, and that the valve-gear, in respect to distribution of the steam, is equal to those already examined. Heusinger's construction is,

without doubt, very ingenious, but too complicated; the remark made at the end of the investigation of Gooch's valve-gear, viz. that the constant lead is purchased at too high a price, is even more applicable here.

CHAPTER V.

Pius Fink's Link-motion.

Description of the Valve-gear.

This valve-gear, which is represented by Fig. 16, Pl. IV., and for which the inventor took a patent as long ago as April 1857, is the simplest link-motion in existence, and has often been applied. O is the driving-axle, O R the crank, and D an eccentric with an angle of advance of 90°, whose sheave is directly and immovably fastened to the link C C. A sustaining arm G Q, which swings round the fixed pin G, is jointed to this link at the point Q, so that the point Q moves back and forth almost exactly on the centre line of the stroke O B, while oscillating movements of the link take place round this point simultaneously with the turning of the axle.

The radius-rod M B_1, which is connected at B_1 with the valve-spindle, is moved up and down in the link C C by means of the bell-crank lever E K L (Fig. 16, Pl. IV.), with which it is connected by the lifting-link E T. The engine then runs in one or the other direction according as the sliding-block M is placed below or above the dead point J of the link; moreover, the distance J M governs the grade of expansion.

This valve-gear has especially been used for stationary engines working with variable expansion but always running in the same direction. For such cases only one-half of the link is constructed (see Fig. 17, Pl. IV.) and the sliding-block M is then fixed at any required point M of the link by means of a screw. This difference, viz. whether the shifting of the radius-rod is effected through a bell-crank by means of the suspension-rod E T, as in Fig. 16, or whether

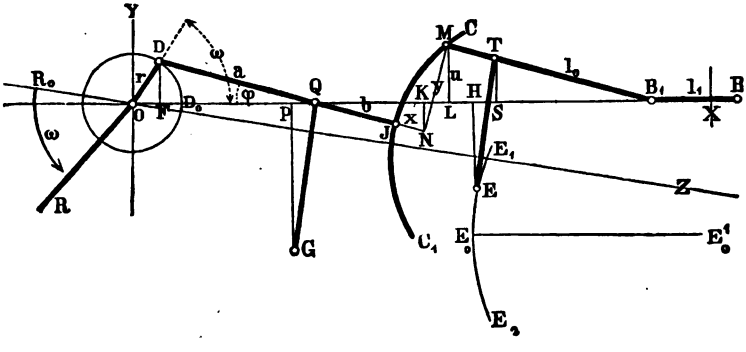
the point M is fixed in the link by a screw, must be taken into consideration during the theoretical investigation, for the movement of the valve will be different in the two cases; in the first arrangement the slide-block M will slip up and down in the link between certain limits, while in the second it will not slip.

Theory of Pius Fink's Link-motion.

(*a.*) *Determination of the Movement of the Valve.*

We shall at first investigate the arrangement shown in Fig. 16, Pl. IV., and shall suppose the radius-rod to be moved up and down by means of a bell-crank lever. If in Fig. 39, O B is the direction of the valve-face and O Z the centre line of the cylinder, the crank

Fig. 39.



stands at a dead point when in the position O R_0 and at this instant O D_0 indicates the line of eccentricity r. The angle of advance is therefore Y O D_0 = 90°; and if the crank is moved in the direction of the arrow through the angle ω, the eccentricity will move to the position O D, and all other parts of the valve-gear represented in the figure correspond to this position.

We will suppose that the point Q, which is situated on the connecting line D J between the centre D of the eccentric and the dead point J of the link, moves back and forth exactly on the line of motion O B; at least we may neglect the variations which will be caused if we take the sustaining arm Q G very long and its fulcrum

G so that the chord of the arc which the point Q describes is parallel to O B. Moreover, in the following calculations we shall represent the two parts D Q and Q J of D J by a and b respectively, and the inclination D Q O of this line to the direction of stroke, when crank-angle $= \omega$, we shall represent by ϕ. We shall again call the whole length of link $C C_1 = 2c$, and for the present suppose the link to be an arc whose radius is equal to any length ρ. Let the point M which represents the position of the sliding-block for the corresponding position of the crank have the co-ordinates $J N = x$ and $N M = y$, the dead point J as the origin of co-ordinates and D J as the axis of abscissa. Next let $B_1 M = l$ represent the length of the radius-rod and $B_1 B = l_1$ the length of the valve-stem measured to the centre of the valve. Now if the suspension-rod E T is long, and if its fixed point E is chosen correctly, the point T, and therefore the end M of the radius-rod, i. e. the sliding-block in the link, will move almost parallel to the centre line of motion O B, and the normal distance $M L = u$ will be a constant one for a certain position of the bell-crank lever E K L (Fig. 16, Pl. IV.). In all valve-gears hitherto examined we represented the distance J M of the slide-block from the dead point of the link by u; but in the present case it is more correct to make the distance $M L = u$, because we thus show in the formulas the influence of the slip of the slide-block M in the link upon the movements of the valve.

We must now determine the distance of the centre of the valve B from the centre C of the axle for the position of the crank $R_0 O R = \omega$. If we drop (Fig. 39) from the points D and N on O B the perpendiculars D F and N K, we have, as will be seen from the figure,

$$O B = O F + F Q + Q K + K L + L B_1 + B_1 B,$$

or, as will be seen at once,

$$O B = r \cos \omega + a \cos \phi + (b + x) \cos \phi + y \sin \phi + \sqrt{l^2 - u^2} + l_1. \qquad (93)$$

There exists in addition the relation,

$$y \cos \phi = u + (b + x) \sin \phi; \qquad (94)$$

and as the link is to be curved to radius ρ, we have

$$y^2 = x\,(2\,\rho - x),$$

or, when the radius ρ is proportionately large, and consequently the arc of the link flat, we have more simply,

$$y^2 = 2\,\rho\,x. \tag{95}$$

Finally, we get from the calculation of the length D F,

$$r \sin \omega = a \sin \phi. \tag{96}$$

The four equations above given comprise the entire theory of the present valve-motion; for if the quantities x, y and ϕ are eliminated, by means of the last three equations, from equation (93), O B is then only a function of ω and u, and the ascertaining of this relation is the very object of the investigation. The exact calculation would lead to very complicated formulas, but we may with sufficient correctness proceed approximately as follows. In equation (93), when the eccentric-rod is long and the link proportionately short, i. e. when l is large and u small, we may write for the quantity under the radical,

$$\sqrt{l^2 - u^2} = l - \frac{u^2}{2\,l}.$$

Substituting also in equation (93) the value of y obtained from equation (94), the former after reduction becomes

$$\mathrm{O\,B} = r \cos \omega + a \cos \phi + \frac{b + u \sin \phi}{\cos \phi} + \frac{x}{\cos \phi} + l + l_1 - \frac{u^2}{2\,l}.$$

The value of x may be calculated from equation (95), and as this value is of itself very small, we can obtain the value of y from equation (94), by putting there $x = 0$; then follows from (94),

$$y = \frac{u + b \sin \phi}{\cos \phi},$$

and then from (95),

$$x = \frac{(u + b \sin \phi)^2}{2\,\rho \cos^2 \phi},$$

and by substituting in the last equation for O B,

$$\mathrm{O\,B} = r\cos\omega + a\cos\phi + \frac{b + u\sin\phi}{\cos\phi} + \frac{(u + b\sin\phi)^2}{2\,\rho\cos^3\phi} + l + l_1 - \frac{u^2}{2\,l}.$$

Finally the angle ϕ must be expressed in terms of ω; from equation (96) follows:

$$\sin\phi = \frac{r}{a}\sin\omega,$$

$$\cos\phi = \sqrt{1 - \frac{r^2}{a^2}\sin^2\omega},$$

or, when a is large enough in proportion to r and the radical is replaced by a series in which all powers higher than the second are neglected,

$$\cos\phi = 1 - \frac{r^2}{2\,a^2}\sin^2\omega,$$

then follows, approximately,

$$\frac{1}{\cos\phi} = 1 + \frac{r^2}{2\,a^2}\sin^2\omega$$

and

$$\frac{1}{\cos^3\phi} = 1 + 3\,.\,\frac{r^2}{2\,a^2}\sin^2\omega.$$

Using these values in the last equation for O B and neglecting all quantities which contain powers of $\frac{r}{a}\sin\omega$ higher than the second, we get finally,

$$\mathrm{O\,B} = r\cos\omega + \frac{r\,u}{a}\left(1 + \frac{b}{\rho}\right)\sin\omega + a + b + l + l_1 + \frac{u^2}{2\,\rho} - \frac{u^2}{2\,l} - \frac{1}{2}\frac{r^2}{a^2}\left(a - b - \frac{b^2}{\rho} - 3\,\frac{u^2}{2\,\rho}\right)\sin^2\omega. \quad (97)$$

According to this formula the distance of the valve centre from the centre O of the axle may be calculated for a certain position u of the radius-rod and for any position ω of the crank.

If the crank stands at one of the dead points, and therefore $\omega = 0$, this distance will be

$$r + a + b + l + l_1 + \frac{u^2}{2\rho} - \frac{u^2}{2l}.$$

If the crank stands at the other dead point, and therefore $\omega = 180°$, this distance is

$$-r + a + b + l + l_1 + \frac{u^2}{2\rho} - \frac{u^2}{2l}.$$

As the valve is to be set so as to have equal lead, the mean of these two values gives the distance O X of the centre of motion of the valve from the centre O of the axle. This distance is

$$\mathrm{OX} = a + b + l + l_1 + \frac{u^2}{2\rho} - \frac{u^2}{2l}.$$

But the centre X must be the same for every position of the radius-rod; hence O X must be independent of u, and this is the case when $\rho = l$; from this follows the rule:—

In Fink's valve-gear the link must be curved to an arc whose radius is equal to the length of the radius-rod $\mathrm{B_1 M} = l$.

Putting now into the equation for O X, as well as in equation (97), $\rho = l$, we finally get from the formula

$$\xi = \mathrm{OB} - \mathrm{OX},$$

the movement of the valve

$$\xi = r\cos\omega + \frac{ru}{a}\left(1 + \frac{b}{l}\right)\sin\omega$$
$$- \frac{1}{2}\frac{r^2}{a^2}\left(a - b - \frac{b^2}{l} - 3\,\frac{u^2}{2l}\right)\sin^2\omega, \qquad (98)$$

and if we may neglect the last quantity, which represents the "missing quantity," on account of its very small value, as is always possible when there is a good distribution of the steam, we get the simple equation

$$\xi = r\cos\omega + \frac{ru}{a}\left(1 + \frac{b}{l}\right)\sin\omega, \qquad (99)$$

which in form is the same as those obtained for all valve-motions hitherto examined.

The results above obtained are based upon the arrangement shown in Fig. 16, Pl. VI., or Fig. 39; if however, the sliding-block M is *not* kept by means of the rod E T parallel to the centre line of motion O B, and therefore is not forced to move up and down in the link, but is fixed in the link at the point M by a screw, the results above obtained cannot be employed without modification; but there is no difficulty in at once transforming the formulas, so that they may correspond to the altered conditions. M L $= u$ is then no longer constant for a certain degree of expansion (Fig. 39), but this is now the case with the length J M, which may be represented correctly enough by y. But according to equation (94) we have

$$u = y \cos \phi - (b + x) \sin \phi,$$

and if we put here, as above,

$$x = \frac{y^2}{2\rho} = \frac{y^2}{2l},$$

as well as

$$\sin \phi = \frac{r}{a} \sin \omega,$$

and

$$\cos \phi = 1 - \frac{1}{2}\frac{r^2}{a^2} \sin^2 \omega,$$

then follows

$$\frac{r u}{a} \sin \omega = \frac{r y}{a} \sin \omega - \left(b + \frac{y^2}{2l}\right)\frac{r^2}{a^2} \sin^2 \omega,$$

as well as

$$\frac{r^2 u^2}{a^2} \sin^2 \omega = \frac{r^2 y^2}{a^2} \sin^2 \omega.$$

Using these two equations for eliminating from equation (98) the value of u, we get, after a few reductions, for the present case

$$\xi = r \cos \omega + \frac{r y}{a}\left(1 + \frac{b}{l}\right) \sin \omega$$
$$- \frac{1}{2}\frac{r^2}{a^2}\left(a + b + \frac{b^2}{l} - \frac{y^2}{2l}\right) \sin^2 \omega; \qquad (100)$$

this formula differs from equation (98) only in the "missing quantity"; when this quantity is small enough the last equation reduces to equation (99) with the difference that here y represents the

distance between the sliding-block M and the dead point J of the link, while in equation (99) u represents the normal distance of the sliding-block M from the centre of motion O B, i. e. the distance M L.

(b.) *The Curve of Centres.*

If Fink's valve-motion is such that the "missing quantity" may be neglected, then from what was said concerning the curves of centres of the other valve-motions we know that the co-ordinates of the centre of the valve circle corresponding to the position u of the radius-rod can be obtained from the formulas:

$$\frac{1}{2}\,\mathrm{A} = \frac{1}{2}\,r$$

$$\frac{1}{2}\,\mathrm{B} = \frac{1}{2}\,\frac{r\,u}{a}\left(1 + \frac{b}{l}\right).$$

Now, as $\frac{1}{2}$ A is constant for all valve circles, their centres are situated above each other; the "curve" of centres is therefore, as in the valve-motions of *Gooch* and *Heusinger von Waldegg*, a *straight line*, which is perpendicular to the line of stroke. A further investigation of the present question would only be a repetition of what has already been given in connection with the valve-motions last mentioned.

(c.) *The Suspension of the Link and of the Radius-rod.*

We shall first discuss the suspension of the link. This is effected by means of the sustaining arm G Q, which swings round the fixed point G (Fig. 39). The theoretical investigations respecting the movement of the valve supposed that the point Q moved backwards and forwards on the line of motion O B; in order to fulfil this condition as nearly as possible, the link G Q must be made as long as possible, and the centre G must be situated so that the chord of the arc in which the point Q really moves is parallel to the centre line of motion. The abscissa O P of the centre G is obtained by this last requirement. We find for any crank-angle ω the distance O Q (Fig. 39),

$$\mathrm{O\,Q} = \mathrm{O\,F} + \mathrm{F\,Q} = r \cos \omega + a \cos \phi.$$

Now if the crank stands at one of its dead points so that $\omega = 0$, and also $\phi = 0$, the corresponding value of O Q is

$$r + a.$$

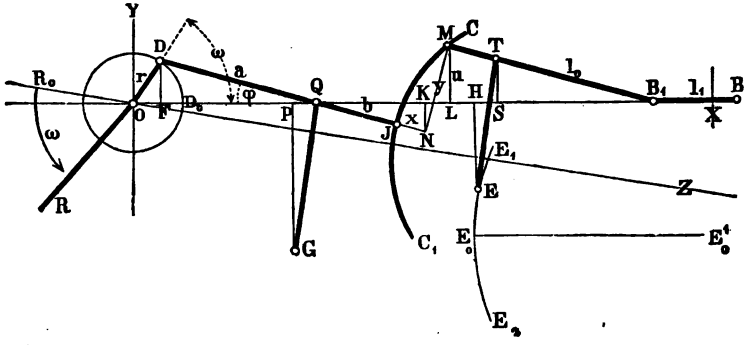
If the crank, however, stands at the second dead point, so that $\omega = 180°$, then again $\phi = 0$, and it follows that the value of O Q is

$$-r + a.$$

The mean of these two values is simply a. The point P, below which the fixed centre G must be placed at the distance G P, is therefore at the distance O P = D Q = a from the centre O of the axle.

As regards the suspension of the radius-rod, the problem is to keep the sliding-block M (Fig. 39) at a constant distance M L = u

FIG. 39.



from the line of motion O B. This condition is best fulfilled when the point T, at which the suspension-rod E T is fastened to the radius-rod, moves in an arc the chord of which is parallel to the centre line of motion, and when the radius of this arc, i. e. the length l_2 of the arm E T, is taken as large as possible. The abscissa O S of the point T for a certain position u of the sliding-block M and crank-angle ω, is now easily determined as follows:—

We have

$$O S = O B - B B_1 - B_1 S$$

and

$$B_1 S : B_1 T = B_1 L : B_1 M.$$

If we put $B_1 T = l_0$, and, as before, the length of the radius-rod $B_1 M = l$, the above proportion gives

$$B_1 S = \frac{l_0}{l} \sqrt{l^2 - u^2};$$

or approximately

$$B_1 S = l_0 - \frac{l_0 u^2}{2 l^2}.$$

Representing the length of the valve-stem $B_1 B$ by l_1 (measured to the centre of the valve), and using in the equation for O S the equation (97), neglecting there the last term (the "missing quantity") and substituting l for ρ, we get

$$O S = r \cos \omega + \frac{r u}{a}\left(1 + \frac{b}{l}\right) \sin \omega + a + b + l - l_0 + \frac{l_0 u^2}{2 l^2}.$$

If the crank stands at one of the dead points, so that $\omega = 0$, the corresponding value of O S is

$$r + a + b + l - l_0 + \frac{l_0 u^2}{2 l^2}.$$

If the crank is at the other dead point, so that $\omega = 180°$, the value of O S becomes

$$- r + a + b + l - l_0 + \frac{l_0 u^2}{2 l^2}.$$

The mean of these two expressions gives that central position of the swinging point T for which the arm E T has to be vertical; and the abscissa $O H = x$ of the point E for the corresponding position u of the sliding-block M is therefore

$$O H = x = a + b + l - l_0 + \frac{l_0 u^2}{2 l^2}. \qquad (101)$$

The ordinate $H E = y$ of this point is, however,

$$H E = E T - S T.$$

Substituting l_2 for the length of the arm E T, and considering the proportion

$$S T : M L = B_1 T : B_1 M,$$

or

$$S T : u = l_0 : l,$$

we get also

$$H E = y = l_2 - \frac{l_0}{l} u. \qquad (102)$$

The co-ordinates x and y of the point E are now calculated by the aid of the two equations (101) and (102) for every position u of

the sliding-block; the course of the curve $E_1 E E_2$ in which this point E must move up and down, so that the guiding of the radius-rod may be as perfect as possible, is thus easily determined.

For the dead point of the link, i. e. for $u = 0$, this turning point falls at E_0, and the co-ordinates $x = x_0$ and $y = y_0$ for this point are therefore

$$x_0 = a + b + l - l_0, \tag{103}$$

$$y_0 = l_2. \tag{104}$$

The position of this point E_0 may now be easily ascertained. Combining equations (101) and (102), we get

$$x - x_0 = \frac{l_0 u^2}{2 l},$$

$$y_0 - y = \frac{l_0}{l} u,$$

and, by eliminating u, the equation of the curve $E_1 E_0 E_2$,

$$(y_0 - y)^2 = 2 l_0 (x - x_0),$$

and this is the equation of a parabola whose parameter is $2 l_0$, whose principal axis $E_0 E_0^1$ is parallel to the line of motion O B, and whose arc (for $2 l_0$ will always be large) can again be replaced by a circular arc of the radius l_0.

We get thus, with reference to the suspension of the radius-rod, the following practical rule :—

Choose the length l_2 of the suspension-rod E T as long as possible, calculate the co-ordinates of the point E_0 according to the equations (103) and (104), and draw through this point the line $E_0 E_0^1$ parallel to the line of motion O B. From E_0^1 describe now, with the length $B_1 T = l_0$ as radius, a circle $E_1 E_0 E_2$ which passes through E_0; this then is the arc in which the fulcrum E of the suspension-link has to be moved up and down in order to produce a variable expansion. The valve-gears on this system, as executed at the present time (compare Fig. 16, Pl. IV.), are not constructed according to this rule, for the arm E K of the bell-crank lever E K L is taken shorter than the part $B_1 T$ of the radius-rod. But a simple change in the design would easily effect the required movement of the point E.

Practical Application of the Diagram.

If we neglect the "missing quantity," the movement of the valve in this valve-gear, according to equations (98) and (100), is

$$\xi = r \cos \omega + \frac{r u}{a}\left(1 + \frac{b}{l}\right) \sin \omega,$$

the different letters having the same signification as those given on p. 154. Hence, if the coefficients of cos. ω and sin ω are determined, the valve circles for different values of u may be drawn; and, on account of u appearing in the equation, a diagram for this valve-gear is obtained which is of the *same* form as that of the valve-gear by *Gooch* (p. 119) and that by *Heusinger von Waldegg* (p. 149). It seems, therefore, unnecessary to explain the application of the above formula and the construction of the diagram by a special example, as we should simply have to repeat what has already been stated. But the peculiarities of Fink's valve-gear may be shown here by comparing it with those previously examined.

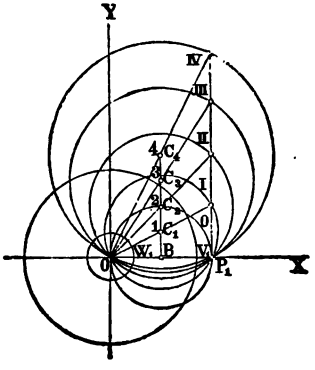
We obtained for the two methods of moving the sliding-block in the link represented in Figs. 16 and 17, Pl. IV., two different formulas (98) and (100) for the movement of the valve; these formulas differ in the "missing quantity," and in the fact that in one formula y or u represents the distance of the sliding-block M from the centre line of motion (M L, Fig. 39), and in the other the distance of the block M from the dead point J of the link (or, with sufficient exactness, the part M N in Fig. 39); in the first case, the sliding-block moves for a certain distance up and down in the link, while in the other case it is fixed. But the fact that the same formula for the movement of the valve under a different signification of u is obtained in both arrangements by neglecting the "*missing quantity*," leads to the conclusion that the two parts M L and M N (Fig. 39) ought not to differ much from each other. But this condition is fulfilled, as an examination of the figure shows, when the eccentricity O D $= r$ is taken small, and the distance D Q $= a$ as large as possible. The "missing quantities" themselves indicate that the ratio $r : a$ must be as small as possible. A further examination of the figure shows that the required condition will be fulfilled

by choosing the part Q J $= b$ as small as possible; it therefore seems best, as it is easiest, to choose the value $b = 0$, i. e. to connect the link with the sustaining arm G Q at the dead point J. We may add to these rules the condition already stated during the development of the formulas, namely, to choose the length of the radius-rod, i. e. the radius of the link, as large as possible.

Now, supposing the above-given conditions to be fulfilled as far as possible for a certain case, the question may arise whether *Fink's* valve-gear, on account of its great simplicity, is able to replace, especially on locomotives, the valve-gears by *Stephenson, Gooch,* and *Allan.*

In order to answer this question let us solve the problem of designing a valve-gear on *Fink's* system, which has to produce *exactly the same* distribution of the steam as the valve-gear on *Gooch's* system, chosen for the problem on p. 119.

Fig. 40.



We found there for the last (fourth) grade of expansion, i. e. for $u = c$, and for the dimensions given in that problem, the co-ordinates of the valve circle (Fig. 40):

$$\mathrm{O\,B} = 0{\cdot}014^{\mathrm{m}}\ (0{\cdot}55'')$$

and

$$\mathrm{B\,C_4} = 0{\cdot}027^{\mathrm{m}}\ (1{\cdot}06'').$$

In the Fink's valve-gear assumed, let $b = 0$ and the radius-rod take its extreme position, i. e. $u = c$ (c representing the half-length of the link), then (for the diagram Fig. 40) from preceding propositions and the equation

$$\xi = \mathrm{A}\cos\omega + \mathrm{B}\sin\omega = r\cos\omega + \frac{r\,u}{a}\sin\omega,$$

we have

$$\mathrm{O\,B} = \frac{1}{2}\mathrm{A} = \frac{1}{2}r$$

and

$$\mathrm{B\,C_4} = \frac{1}{2}\mathrm{B} = \frac{1}{2}\frac{r\,c}{a},$$

and substituting the given numerical values, we get

$$r = 0 \cdot 028^{m} \; (1 \cdot 1'')$$

and

$$\frac{c}{a} = 1 \cdot 93.$$

From this follows, that the full length 2 c of the link in *Fink's* valve-gear must in the present case be nearly *four times* the length D Q = a; the same distribution of the steam could, therefore, only be produced by a *very* long link, as the length a had to be taken as large as possible; but the supposition of a long link contradicts the supposition made when developing the principal formulas, where the distance u, and consequently c, was assumed to be small in comparison with the length of the radius-rod l. These remarks suffice to show that *Fink's* valve-gear is only applicable for smaller periods of admission and high expansion, and that it is therefore inferior to *Stephenson's*, *Gooch's* and *Allan's* gears.

But the "missing quantities" of the formulas (98) and (100) indicate that *Fink's* valve-gear fulfils the conditions required for a good valve-gear less satisfactorily than the others. A closer examination of actual constructions has shown that the "missing quantity" in *Fink's* valve-gear has a proportionally greater influence than in other gears; consequently more disadvantageous deviations from the correct movement of the valve appear.

In the arrangement with slipping-block (Fig. 16, Pl. IV.) we have, according to formula (98), the "missing quantity"

$$-\frac{1}{2}\frac{r^2}{a^2}\left(a - b - \frac{b^2}{l} - 3\,\frac{u^2}{2\,l}\right)\sin^2\omega.$$

The same quantity for the other arrangement with fixed block (Fig. 17, Pl. IV.), according to formula (100), is

$$-\frac{1}{2}\frac{r^2}{a^2}\left(a + b + \frac{b^2}{l} - \frac{y^2}{2\,l}\right)\sin^2\omega.$$

The "missing quantity" varies with the crank angle ω; it is *nil* only for $\omega = 0$ and $\omega = 180°$, i. e. when the crank passes through the dead points; it is, however, greatest for $\omega = 90°$ and $270°$, and for the present valve-gear this is especially unfavourable. As this

valve-gear is principally used for high expansion, i. e. for short cut-offs, the "missing quantity" will exercise its greatest influence at just those crank angles which generally correspond to the beginning of the expansion, that is, at just the moments when the variations of the movement of the valve ought to be avoided.

Stephenson's, *Gooch's*, and *Allan's* valve-gears show more favourable results in this respect. The "missing quantity" for these three constructions was of the same form:

$$+ \frac{r^2}{2\,l}\left(\cos 2\,\delta \sin \omega \pm \frac{u}{c} \sin 2\,\delta \cos \omega\right) \sin \omega.$$

(See pp. 73, 111 and 128.)

This value in these cases becomes *nil* not only for two, but for *four* positions of the crank, not only for $\omega = 0$ and 180°, but also for the angles which are obtained from the formula

$$\tan \omega = \mp \frac{u}{c} \tan 2\,\delta;$$

for the particular case of open rods and the highest grade of expansion, i. e. for $u = c$, we get $\omega = 180° - 2\,\delta$ and $\omega = 360° - 2\,\delta$.

Differentiating the "missing quantity" with respect to ω, we determine the crank angle at which this quantity is a maximum from the formula

$$\tan 2\,\omega = \pm \frac{u}{c} \tan 2\,\delta,$$

which gives four values. The "missing quantity" becomes a maximum for open rods (upper sign) and the highest grade of expansion when the crank angles are

$$\omega = 90° - \delta;\ 270° - \delta;\ 180° - \delta;\ 360° - \delta.$$

Hence, the greatest variation takes place when the valve is at the greatest distance from the centre of its stroke, and also when it passes through the centre of its motion; but these are the positions at which the variations evidently influence the distribution of the steam least.

All these observations show that *Fink's* valve-gear is most decidedly inferior to the other valve-gears, and that it can scarcely be

applied to locomotives. In many cases, however, it is applied with advantage to stationary engines on account of its simplicity.* In the 'Civilingenieur,' Bd. 19, 1873, p. 222, the engineer *Hunäus* has shown analytically that the *curved link* in Fink's valve-gear could be replaced by a *straight link*, provided the link were lowered vertically when the radius-rod was raised, and *vice versâ*, after the manner of Allan's valve-gear. A still more general treatment of the problem is suggested by the comparison of *Stephenson's*, *Gooch's*, and *Allan's* valve-gears given on p. 134.

* ["*Prof. Zeuner's* views of the unsuitableness of the 'Fink' link as a *reversing*-gear are correct, and for the reasons he gives. But the use of the *half*-link in the Porter-Allen engines (it is in connection with these engines that the Fink motion was first brought into extensive use) shows that when compression is judiciously applied, then the cut-off points may be made practically 'symmetrical' up to the half-stroke *at the expense of a variable lead*, the variation of which is in opposite directions for the two strokes. Before cushioning was used in these engines the variability of the lead made it almost impossible to obtain quiet running under varying conditions."]

APPENDIX TO FIRST PART.

On the Counter-pressure of Steam in Engines with Reversing-gear.

All the above investigations on the distribution of steam by link-motions started with the supposition that the link-motion worked in fore-gear when the engine was running forward, and in back-gear when the engine was running backward; in other words, it was assumed that the steam acted in the locomotive cylinder so as to cause a progressing motion, or relatively, an acceleration of the motion.

But there is one case now of special interest, and which we will call the "counter-pressure of the steam," which can arise in all engines provided with reversing-gear; we shall therefore in the following say a few words about this case, as our diagram will explain it without requiring any further alterations or additions. *Lechatelier's* proposition to use counter-pressure of steam as a brake for stopping railway trains, has given this question greater importance.

If we suppose a locomotive to be running at a high speed, and the valve-motion suddenly reversed so that the valve-gear works in a reversed manner, for example so that the valve-gear distributes the steam during the forward course of the engine in such manner as would cause the engine to run backwards were a new start being made, very peculiar effects will evidently occur. This much will be at once seen, that the speed of the locomotive and train will be diminished, and that the steam will act against the piston. In order to explain the effects which occur in this case, we will suppose a locomotive with Stephenson's valve-gear; the chief positions of the piston for the different (four) grades of expansion are represented by the upper part of Figs. 8 and 10, Pl. II., Fig. 8 representing the valve-gear with open rods, and Fig. 10 that for crossed ones. Now suppose the engine to be running forwards, the valve-gear in fore-gear and the piston travelling away from the main driving-axle, as indicated by the arrow (Figs. 8 to 10); then, from what has preceded we know that these four positions of the piston *a*, *b*, *c*, and *d* correspond to the following occurrences:—

a	Beginning	of the	expansion,	in cylinder end	nearest to	driving-axle.
b	„	„	compression,	„	farthest from	„
c	„	„	exhaust,	„	nearest to	„
d	„	„	admission,	„	farthest from	„

But the representation may become clearer by designating the distances travelled by the piston.

The occurrences in cylinder end *nearest* to driving-axle are

Distance H *a* admission of the steam (admission);
„ *a c* expansion of the steam;
„ *c* K release of the steam (pre-exhaust).

But the occurrences in cylinder end *farthest from* driving-axle are

Distance H *b* release of the steam;
„ *b d* compression of the steam;
„ *d* K admission of the steam (pre-admission).

By means of these values, which have been obtained from the diagram, it is now easy to imagine the corresponding indicator diagram and to judge of the work performed by the steam.

Let us investigate the *counter-effect of the steam*, and accordingly suppose that the engine runs forwards while the valve-gear at one of the four grades of expansion is in *back-gear; the order of the four principal positions of the piston will then be simply reversed*, or the arrow which indicates the stroke of the piston in Figs. 8 and 10 must be drawn in the opposite direction.

If we therefore suppose the piston to travel from K towards H (Figs. 8 and 10, Pl. II.), the four principal positions of the piston will correspond with the following occurrences:—

d	Beginning	of the	expansion,	in cylinder end	farthest from	driving-axle.
c	„	„	compression,	„	nearest to	„
b	„	„	exhaust,	„	farthest from	„
a	„	„	admission,	„	nearest to	„

Or, if we again express the occurrences in terms of the *distances travelled by the piston*, we have

In cylinder end *farthest from* driving-axle:

Distance K *d* admission,
„ *d b* expansion,
„ *b* H exhaust (pre-exhaust).

Occurrences in cylinder end *nearest to* driving-axle:

Distance K *c* exhaust (more correctly, cylinder space communicates with the exhaust-port);
„ *c a* compression;
„ *a* H admission (pre-admission).

Thus it is easy to get an idea of the effect of the steam in the cylinder, and to imagine the corresponding indicator diagram. If we examine more minutely the occurrences in cylinder end farthest from driving-axle, we shall find that fresh steam enters into the cylinder while the piston travels

the distance K *d*, which is particularly short at the last grade. At *d* the cutting-off, and consequently the expansion, begins, lasting until the piston arrives at *b*; the distance *d b* during which expansion takes place, is now so large in comparison with the distance K *d*, during which admission of the steam takes place, especially at the farthest grade of expansion, that the pressure of the steam in the cylinder generally sinks almost to or below the atmospheric pressure. The exhaust-port, which for the present is supposed to communicate directly with the open air, opens when the piston arrives at *b*. If at this instant the pressure of the steam in the cylinder is below that of the atmosphere, this inequality will disappear by the outside atmospheric air entering the cylinder (no steam is therefore admitted); but if the pressure of the steam in the cylinder has not fallen, in consequence of the expansion, to or below the atmospheric pressure, then enough steam escapes at the moment of opening the exhaust-port, to bring the remaining steam to atmospheric pressure.

The piston, when it has travelled the distance *b* H, has reached the end of its stroke, and during this whole distance the outside air is *drawn in* through the common exhaust-port, so that when the piston has arrived at *b*, the whole cylinder is filled by the atmospheric air and the small volume of steam which entered while the piston travelled the distance K *d*. Exactly the same occurrences will also take place in cylinder end nearest driving-axle while piston is returning to the position K after it has travelled through the distance H K. Now let us suppose that the piston has returned to the point K, and let us again follow it during its travel from K to H, but this time direct our attention to the occurrences which take place in cylinder end nearest driving-axle (Figs. 8 and 10, Pl. II.). The whole space of the cylinder, at end nearest driving-axle, is filled with atmospheric air and a little steam. The exhaust-port is still open for the comparatively short distance K *c*, and a small portion of the air will therefore be again driven out, while the remainder will be slightly compressed; but the exhaust-port shuts when the piston arrives at *c*, and the compression begins here, and lasts until the piston occupies the position *a*.

The port for the admission of the steam opens at *a* and remains open during the remaining distance *a* H of the stroke of the piston. The steam from the boiler will mix at *a* with the compressed air, and the whole mass of air and steam will be pressed into the boiler, when nearly the full boiler pressure must be overcome. We see, therefore, that the steam-engines of a locomotive, subjected to the "counter-effect of the steam," no longer act as prime movers, but as "air-pumps"; they are driven by the work, which exists in the form of *vis viva* in the moving train, and nearly a full volume of atmospheric air is drawn into each cylinder at each stroke of the piston, and then compressed and driven into the boiler; four cylinder volumes of

air will therefore enter the boiler of a locomotive at every revolution of the driving wheels.

It is easy to understand that the "counter-effect of the steam" in locomotives causes the pressure of the steam in the boiler to rise quickly, but that is not the principal reason why "the counter-effect of the steam" has hitherto been applied as a brake only in emergencies, for another disadvantage of a more serious character is connected with this method of using steam. In locomotives the exhaust-port does not open directly into the air, but communicates by means of the exhaust-pipe with the interior of the smoke-box. Consequently, suction will not fill the cylinders with pure air, but with the products of combustion from the smoke-box, which have not only a very high temperature of 500°–600° F., and upwards, but also carry with them a great many unconsumed parts of the fuel. The hurtful influence upon the engine and boiler of continually using the counter-pressure of the steam will therefore be at once understood. Nevertheless, engineers have always had sufficient reason to regret that the driver is prohibited from applying this simple and powerful plan of stopping trains, particularly as the retarding of trains by means of friction-brakes is to be considered, theoretically, quite as imperfect a practice, for the full *vis viva* of the trains corresponding to the diminution of velocity is entirely destroyed.

It is not surprising that under such circumstances *Lechatelier's* proposition soon met with great approval and was frequently applied. According to this proposition, when the reversal is effected, the blast-pipe is to be closed, and the exhaust-port connected by a branch pipe with the steam and water space of the boiler; small, controllable quantities of steam and *water*, instead of air and the products of combustion, are thus allowed to enter into the cylinders while the pistons are engaged in suction. Water is admitted, in order to prevent a superheating of the steam during the compression, i. e. in order to prevent the pistons and valves from running dry.

But experience and closer examination of the diagram show that *Lechatelier's* method, especially at a high speed of the engine, is not powerful enough in many cases. At a high velocity the piston will, in consequence of the gradual opening of the port for the admission of the counter-steam, travel a considerable distance before the full counter-pressure acts against the piston. New devices are therefore proposed, namely, the so-called steam repression-brakes; in these there is no reversing; for example, Von Landsee* proposes to fix on the cylinder a second valve, whose eccentric is fastened to the axle with an angle of advance $\delta = 0$. In order

* 'Mémoire sur les différentes méthodes employées pour modérer les vitesses des trains sur pentes et en particulier sur le frein à vapeur, système A. de Landsee.' Mülhouse, 1867.

to stop the engine, the exhaust-pipe is closed, the chief valve-gear is put at a high grade of expansion, and counter-steam admitted through the second valve during the whole stroke of the piston. But the steam repression-brake of Messrs. Krauss and Co., of Munich, appears to accomplish the desired object best.* The blast-pipe is in this case closed towards the outside, but its inside is brought into communication with the steam-space of the boiler; no reversing takes place, but the throttle-valve is closed. The effect thus produced may easily be determined by means of our valve diagram; but this is not the place to follow out this question, and we refer for further information to the interesting pamphlet by Professor Linde: '*Ueber einige Methoden zum Bremsen der Locomotiven und Eisenbahnzüge, insbesondere über die Dampfrepressions-Bremse. Pat. System der Locomotivfabrik Krauss und Comp.' München*, 1868. A thorough and interesting discussion of *Lechatelier's* proposition is given by Prof. *Gustav Schmidt*, in 'Mittheilungen des Architekten und Ingenieurvereins in Böhmen, 1869,' in an article entitled "Anwendung des Gegendampfes bei Locomotiven" (see also 'Zeitschrift des Vereins deutscher Ingenieure,' 1870, Bd. 14, S. 778).

* Described in 'Engineering,' vol. vi., p. 475.—TRANSLATOR.

SECOND PART.

VALVE-GEARS WITH TWO VALVES.

VALVE-GEARS WITH TWO VALVES.

THE valve-motions which we examined in the First Part of this book were constructed in such a manner that the distribution of the steam was effected by means of a single valve; if the movement of this valve is derived from a link, then it is possible, as has been fully explained in the First Part, to reverse the engine and to make it run forwards and backwards with variable expansion. But experience, and an inspection of the diagram given for the link-motions, show in this case a peculiarity in the distribution of the steam; for the higher the expansion, i. e. the earlier the steam is cut off by the *single* valve, the greater the compression of the steam becomes behind the piston; also, the valve then shuts the ports earlier for the release of the steam. The steam, which has to escape into the open air, is confined behind the piston, and the piston in travelling forwards compresses it till the valve opens the port for the admission, an event which generally takes place before the piston has reached the end of its stroke. But the compression of the steam requires power, which the engine under certain circumstances loses. This is the reason why many engineers declare the link-motions with one valve to be bad arrangements for producing expansion, and consider them only useful as arrangements for reversing the engines.

Much, however, may be said against the correctness of this opinion.

First, it is well known that other constructions, for instance the double-valve motions, which compress the steam slightly, if at all, could not replace at the present time the link-motions with one valve. On the contrary, the complicated valve-motions with two valves have been abandoned on the locomotives of several lines, and the simpler valve-motions with one valve have again been applied.

Next, it is to be observed that the locomotive drivers, who certainly have a special interest in making the best use of their valve-motions, mostly use those grades of the link at which the influence of the compression ought to be very disadvantageous.

We might conclude at once from these statements, that an essential loss of power is not, at least under all circumstances, produced by the compression of the steam in consequence of a very early cutting off. But a closer examination of our diagram will at once lead us to the result, that the link-motions with one valve may, under certain circumstances, produce an advantageous distribution of the steam, even for a high degree of expansion.

The upper part of Fig. 8, Pl. II., represents the principal positions of the piston in a valve-gear of the *Stephenson* type; let us examine the positions corresponding to the second grade of expansion. The expansion begins at *a*, the compression at *b*; the steam has at the beginning of the latter a pressure behind the piston a little higher than an atmosphere; the compression lasts until the piston has arrived at the position *d*, and then the exhaust of the steam at once begins. Now, if we imagine that the steam at *d*, in consequence of the compression, has attained a pressure behind the piston just equal to that of the entering steam, then in a certain sense we have produced by compression, fresh steam, which will give out its work at the next stroke of the piston. We may therefore assume that a large part of the work expended on the compression is again given out. That the steam at *d* may have a pressure just or nearly equal to that of the steam in the boiler, it is merely necessary to give to the so-called hurtful space the required volume. This space is without doubt of great importance and exerts a useful influence in link-motions, and the different opinions respecting the effect of the link-motions with one valve, as derived from experience, are perhaps due to the circumstance, that this space is not of the proper size in the different locomotives.

But analysis offers a better explanation of the question here raised. *Reuleaux* first gave in his treatise '*Ueber die Wirkung der Dampfvertheilung bei den Coulissensteuerungen*' (On the Effect of the Distribution of the Steam in Link-motions) (published in the

'Civilingenieur,' vol. iii. page 43) an exact account of the influence of the so-called hurtful space upon the compression; and later, Zeuner in his book, '*Grundzüge der mechanischen Wärmetheorie*' (Principles of the Mechanical Theory of Heat), has fully explained the influence of the compression of the steam and of the hurtful space upon the whole working of the steam-engine, so far as it is possible to do so in the present state of this theory.

But this question is not yet decided, for correct and exact indicator trials are wanted; trials which should be undertaken with special consideration of the requirements demanded by the theoretical investigations.

This is not the place to enter more fully upon the question indicated; we simply wish to call attention to the fact that the valve-motions with two valves are principally indebted for their origin and application to the opinion that the compression of the steam behind the piston, as it occurs in the case of the link-motions with one valve at a high degree of expansion, is disadvantageous, but that this opinion is by no means to be accepted as entirely justifiable.

One of the two valves, the expansion-valve, only effects the cutting off of the steam, and such an arrangement of the different parts of the valve-motion is therefore generally chosen, that this cutting off may take place either sooner or later; or, in other words, the expansion is generally a variable one. The *second* valve, the main valve, regulates only the beginning of the admission, the exhaust, and the lead; it generally has an invariable stroke and small inside and outside laps. This main valve is, in the reversing motions, moved by the link, which then simply serves to effect the reversing, and is only used in its extreme positions.

The disadvantage attending compression, supposing that we can, after what we have said, speak about such a disadvantage, certainly disappears when a special expansion-valve is applied; but now there arises at once the other question, whether the advantage which is expected in consequence of the removal of the compression of the steam, is not partly or perhaps entirely balanced by the considerable loss of power which is connected with the movement of a *second* valve. Only experience and exact trials can give a solution to this question;

but the experience already derived from locomotives indicates that valve-motions with double valves are only applied with advantage at a high pressure of the steam, and *that the link-motions with one valve answer very well for the steam pressures usually employed at the present time.* But as soon as the pressure in the boilers of locomotives is increased to 10 and 12 atmospheres—as has sometimes been done with perfect success—the application of a special expansion-valve will perhaps become indispensable. At the present state of our knowledge, respecting the most advantageous use of the steam power, everything indicates that further progress in the use of steam is principally to be expected by employing steam of as high a pressure as possible, and thus producing a higher, but more perfect, expansion.

A beginning has already been made in locomotives, and as soon as the employment of high-pressure steam shall have become more general in these engines, valve-gears with two valves will receive, perhaps, more attention, and will also be more frequently employed.

We shall therefore examine valve-gears with two valves as thoroughly as we examined the valve-gears with a single valve in the First Part; but we have in this case, as in the previous one, only to answer the question, whether the common forms of valve-gears, with separate expansion-valve, are so designed as to fulfil the conditions which a correct valve-gear must satisfy in order to produce variable expansion. It is chiefly necessary that the steam may be cut off at any position of the crank. The following investigations will show whether this is really the case in the systems employed at the present time, and if not, what degree of expansion may be produced by each system, and what improvements are possible in the different constructions.

The method of investigation will be different in this case from that hitherto employed, in that we shall give the theory and the practical application of the diagram simultaneously. The theoretical discussions are so simple, that every reader acquainted with the elements of mathematics will easily understand them.

There are two kinds of valve-gears with double valves; in the *first* kind there are two steam-chests, one above the other; the

ordinary main valve moves in the lower one and the expansion-valve slides in the upper one on the perforated plate which separates the two steam-chests. The upper steam-chest is in uninterrupted communication with the boiler, and the expansion-valve only regulates the admission of steam to the lower chest.

In the *second* kind of valve-gears there is only one steam-chest, and the expansion-valve slides directly on the back of the main valve which is provided with passages that are alternately opened and closed by the upper valve.

In the following discussions the two kinds will be treated separately, although the valve diagrams of one kind may be easily reduced to those of the other, and although no essential difference exists between them.

SECTION I.

SEPARATE EXPANSION-VALVE SLIDING ON COVER OF MAIN-VALVE CHEST.

CHAPTER I.

GONZENBACH'S CUT-OFF GEAR.

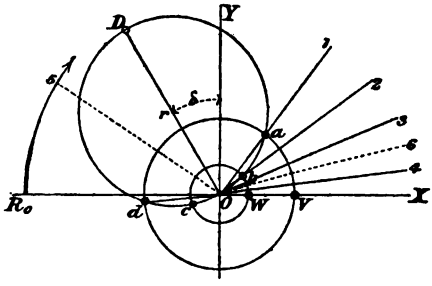
Gonzenbach's valve-gear is shown in Fig. 18, Pl. IV.; both valves, the main valve S as well as the expansion-valve S_0, are shown in their middle positions, though these are positions which they can only occupy *simultaneously* when their eccentric-rods are detached from the eccentrics. The two steam-chests K and K_0 are separated by a partition containing a passage a_0 through which the fresh steam can pass from the upper to the lower steam-chest when the passage is not closed by the expansion-valve S_0 that slides back and forth over the opening; the valve itself is sometimes called a *ported-valve.* In Fig. 18*a* both eccentrics are shown in the positions which they occupy when the crank passes through the first dead point. The eccentricity $OD = r$ of the main-valve eccentric makes the angle $DOY = \delta$ (angle of advance) with the perpendicular OY to the direction of the stroke of the eccentric-rod; the eccentricity $OD_0 = r_0$ makes at the same instant the angle $D_0OY = -\delta_0$ with this perpendicular OY; the following investigation will show that this must be so.

Let us first disregard the action of the expansion-valve, and only consider the action of the main valve, then the distribution of steam takes place as described in the first part of this treatise. It would be unnecessary to discuss this matter again were it not desirable to change the designations for the sake of clearness. When, in the valve-gear in Fig. 18 and Fig. 18*a*, Pl. IV., the eccentricity r and

angle of advance δ are known for the main valve S, and when the inside lap i and outside lap e (the latter being always smaller with than without an expansion-valve) are given, we proceed as follows:

Describe (Fig. 41) the valve circle on $OD = r$ as diameter, then describe circles with O as a centre and $OV = e$ and $OW = i$ as radii, and determine the four intersections $a\,b\,c\,d$ of the three circles; next connect these intersections with O, and the straight lines obtained give the principal crank positions which we will now designate by numbers. When the engine-crank is at the first dead point, the diagram crank is at OR_0; if we suppose the latter to turn in the direction of the arrow and the cylinder to be at the right of Fig. 41, the piston will move away from the crank-shaft (and this motion agrees with that assumed in the arrangement shown in Fig. 18 and Fig. 18*a*, Pl. IV.). Then crank position No. 1 corresponds to the beginning of the expansion in cylinder end nearest crank-shaft, No. 2 to the beginning of compression in cylinder end farthest from crank-shaft, No. 3 to the beginning of exhaust in cylinder end nearest to shaft, and No. 4 to the beginning of admission in cylinder end farthest from shaft. Positions 1 and 4 are the ones of special importance in the following investigations. If we now consider the action of the expansion-valve S_0 (Fig. 18, Pl. IV.) in a general way, we see that this valve simply regulates the admission of steam to the lower valve-chest; for instance, it may be required that the steam be cut off when the crank is in position 5 (Fig. 41); the expansion therefore begins at this position although the main valve does not cut off the steam in the lower valve-chest from the steam in the cylinder till the crank has reached position 1. With *this* arrangement of valves the expansion is a peculiar one. While the crank is passing from position 5 to 1, the steam expands

Fig. 41.



simultaneously in the cylinder and lower valve-chest K, and only while the crank is passing from 1 to 3 does it expand in the cylinder *alone*, for not until position 1 is reached does the main valve separate the two spaces; at position 3 steam begins to exhaust from the cylinder. Now there is a certain crank position 6 at which the upper valve S_0 (Fig. 18, Pl. IV.) reopens the passage in the partition after closing it at position 5. It is evident that it is *absolutely necessary* that position 6 (in Gonzenbach's arrangement) shall fall between positions 1 and 4 of the crank. For if position 6 were in *front* of No. 1, steam would enter the cylinder *twice* during the same stroke, as the expansion-valve would reopen the passage in the partition before the main valve had shut off (at No. 1) the steam space in the cylinder; thus while the crank is passing from the dead point to position 5, steam is admitted to the cylinder, from position 5 to 6 expansion takes place, and from position 6 to 1 steam is again admitted; this is therefore an exceedingly imperfect method of utilising the steam, but nevertheless one which must have frequently occurred in locomotives employing *Gonzenbach's* valve-gear.

Moreover, crank position 6, at which the passage in the partition begins to open, must not fall *behind* position 4; for at the latter position the main valve opens the steam-port at the cylinder end farthest from crank-shaft, in order that the full pressure of fresh steam may act upon the piston at the beginning of its stroke. But this pre-admission would be useless if the passage controlled by the expansion-valve were not already open or at least beginning to open when the main valve opens the steam-port in the cylinder, in other words No. 4 is the latest possible position for No. 6. The angular interval between positions 1 and 4 is usually very small because very small outside laps are assumed for the main valve. Therefore as a rule the choice of position No. 6 is confined within very narrow limits in *Gonzenbach's* valve-gear. We shall see that this is what so greatly restricts the expansion in this valve-gear. After these preliminaries, we may proceed to investigate the action of the expansion-valve; but in order to, at the same time, lay the foundations for the discussions of all other valve-gears with double

valves, and to shorten future investigations, we shall first consider the action of a simple flat valve, and show that the gridiron-valve is only a combination of two separate valves consisting of simple flat plates.

In Fig. 42 and Fig. 43, let A A represent the partition between the two valve chambers, and let a_0 represent the width of the passage in this partition. In both cases let the expansion-valve S_0, which is

FIG. 42.

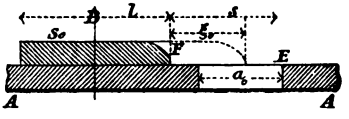


FIG. 43.

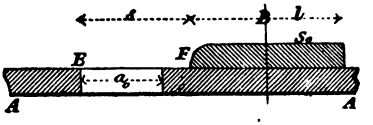


FIG. 42a.

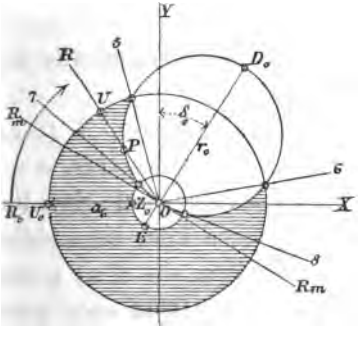
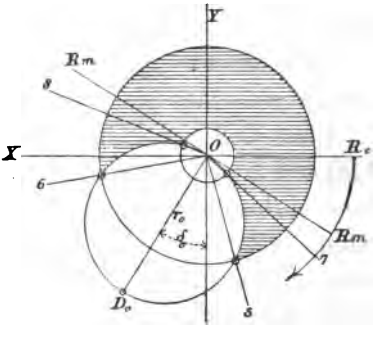


FIG. 43a.



here shown as a simple plate, occupy its middle position at B. In the one case (Fig. 42) this valve stands to the left of the passage, so that the right-hand valve edge F is at the distance s from the farthest edge E of the passage. In the other case (Fig. 43) let the plate be at its middle position, and to the right of the passage. Let us now suppose that in each case the plate is moved by an eccentric $O D_0 = r_0$, which is keyed to the shaft with the negative angle of advance δ_0, as shown in Fig. 18*a*, Pl. IV., the problem before us is simply to determine how far the valve has moved from its middle position when the crank has turned through the angle ω, how much the passage in the partition is open at this instant, and the crank positions at which the passage is closed and reopened.

In an earlier portion of this treatise we found, for a valve whose eccentric had the eccentricity r and angle of advance δ, the valve movement ξ for the crank angle ω to be: $\xi = r \sin(\omega + \delta)$; in the present case the same formula applies, but we must substitute $-\delta_0$ for δ and r_0 for r; the deviation ξ_0 of the expansion-valve from the centre is therefore

$$\xi_0 = r_0 \sin(\omega - \delta_0), \tag{105}$$

and the opening a_x of the passage in the partition at this instant is readily found from Fig. 42 to be

$$a_x = s - \xi_0. \tag{106}$$

The same Fig. 42 shows us that when the valve moves to the *right* the passage is ultimately closed, but when it moves to the left of the middle position the passage remains open. In the arrangement assumed in Fig. 43 just the reverse obtains.

In order to express formulas (105) and (106) graphically we proceed as follows (the references relate to Fig. 42).

Erect O Y perpendicular to the direction of stroke O X (Fig. 42*a*) and lay off the angle $Y O D_0 = \delta_0$ and the distance $O D_0 = r_0$. Now on this distance as a diameter, describe the valve circle, then its radius vectors will give the distance of the valve from its middle position; for example, if the crank has moved from the dead point $O R_0$ through the angle $R_0 O R$, the distance O P will at once give the valve movement ξ_0. When the crank occupies the position $O D_0$, the valve will be farthest from its central position, and when the crank is perpendicular to $O D_0$, i. e. lies in the direction $O R_m$, the valve will occupy its central position. Only one valve circle is shown in Fig. 42*a*, and it shows the distances of the valve to the right of the middle position; the distances to the left are here of no interest, because we already know that, during this part of the motion, the passage is wide open.

In order to complete the diagram and construct equation (106), let us describe a circle from O (Fig. 42*a*) as a centre, and with $O U_0 = O U = s$ as a radius, then we at once get the distance $U P = O U - O P$, i. e. the opening of the passage for this crank position:

$$a_x = U P = s - \xi_0,$$

and this gives us a new diagram which will be employed in all the following valve-gears with double valves.

Through the intersections of the latter circle with the valve circle draw the crank positions 5 and 6; at these positions the opening of the passage in the partition is zero, the cutting off taking place at position 5 and the reopening of the passage at 6. During a rotation of the crank the passage is completely closed while the crank is traversing the distance 5–6, and open while the crank is traversing the distance 6–5.

If we lay off the width a_0 of the passage from U_0 to O, i. e. make $U_0 Z_0 = a_0$, and describe with $O Z_0$ as radius a third circle, its intersections with the valve circle will give us the following. At crank position 7 the opening in the passage begins to diminish, and at 8 is again *completely* open, remaining so until the crank, turning in the direction of the arrow, again reaches position 7. In like manner Fig. 43*a* is the diagram for the valve arrangement shown in Fig. 43; only the lower circle is here employed because the valve covers the passage only while it is at the *left* of the middle position.

But neither of the arrangements shown in Fig. 42 and Fig. 43 can be used alone as an expansion-gear. If we suppose both plates to be moved *simultaneously by the same eccentric*, the arrangement shown in Fig. 42 will act correctly only during the first half of the rotation of the crank; during the second half of the rotation it will have no effect, for steam will be continually streaming through the passage; with the other arrangement just the reverse obtains, this working correctly only while the crank is traversing the second half of its rotation. It is evident that it is only necessary to correctly connect the two cases, and this may be done in various ways; the simplest and, as will appear later, the most imperfect method was that employed by *Gonzenbach*. Before we enter upon this matter, however, the above propositions must receive an important extension, of which frequent use will be made hereafter.

It was assumed in Fig. 42 that the expansion-valve S_0, consisting of a simple plate, was entirely on the left of the passage; moreover, it was tacitly assumed that during the motion of the valve towards the right, its *left* edge did not pass beyond the left edge of the

passage; in order that this latter assumption may be realised, the plate must have sufficient length. Let us designate this length by l, then the figure shows that the valve must be moved through the distance $l + s - a_0$ before the left edge of the valve can stand over the left edge of the passage. But the greatest valve movement is r_0; therefore the assumed case cannot occur, when

$$l + s - a_0 > r_0$$

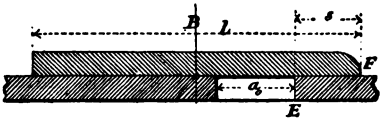
or

$$l > r_0 - (s - a_0), \qquad (107)$$

and this condition is always easily fulfilled. The value of the right member of this inequality can be taken directly from the diagram Fig. 42*a*, in which $O D_0 = r_0$ and $O E = s - a_0$.

We will now discuss the case in which the plate, while in its *middle position*, completely covers the passage, as in Fig. 44. In order that the left edge of the valve may not pass beyond the left edge of the passage when the valve occupies its extreme position on the right, the condition

Fig. 44.



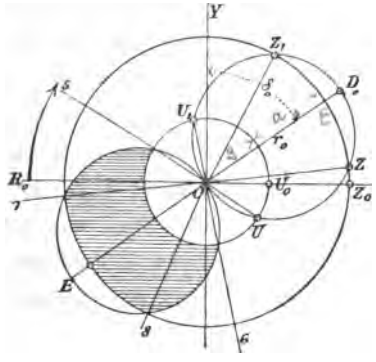
$$l - s - a_0 > r_0$$

or

$$l > r_0 + s + a_0 \qquad (108)$$

must be fulfilled; and that it is fulfilled will be assumed in what follows. (In the diagram Fig. 44*a*, we have $O D_0 = r_0$ and $s + a_0 = O E$, consequently l must be greater than $D_0 E$.) Now the peculiarity of this arrangement consists in the *negative* value of s; let us again describe from O, Fig. 44*a*, a circle $O U_0 = O U = s$ and again find its intersections U and U_1 with the valve circle; but in the present case the intersection U on the *negative* side of the radius vector

Fig. 44*a*.



corresponds to the instant of cut-off; if we prolong U O beyond O, position 5 will correspond to the beginning of expansion and position 6, or the prolongation of the line connecting the intersection U_1 with O, will correspond to the reopening of the passage.

Furthermore if we lay off $U_0 Z_0 = a_0$ = width of passage and describe the third circle from O as a centre and $O Z_0 = s + a_0$ as radius, we obtain the two other intersections Z and Z_1, and connecting these with O and prolonging the lines beyond O, we get crank position 8 at which the passage is fully open and position 7 at which the valve begins to close the passage. If, as may occur in practice, $O Z_0 > O D_0$, i. e. $s + a_0 > r_0$, the two intersections Z and Z_1 will no longer occur; in this case the passage is never fully open even when the valve is at the extreme left of its stroke.

There is another special case to which we may call attention, that in which the valve edge F coincides with the edge E of the passage when the valve is in its middle position. Here $s = o$ and the two positions 5 and 6 coincide in a straight line which is perpendicular to $O D_0$.

A comparison of Fig. 42*a* and Fig. 44*a* clearly shows the difference in the action of an expansion-valve consisting of a simple plate, according as the distance designated by s is taken positive or negative. The case represented in Fig. 44*a* does not occur in *Gonzenbach's* arrangement; the object in treating it here was to deduce a diagram for all cases of the expansion-valve, and thus obtain a basis for the discussion of all other gears with expansion-valves.

As regards *Gonzenbach's* valve-gear, the two cases represented by Fig. 42 and Fig. 43 are combined as represented in Fig. 45. Both plates are shown in their middle position, are arranged symmetrically on both sides of *the same passage*, and are rigidly connected with each other, being driven by the same eccentric $r_0 \delta_0$. Between the two plates there is a rectangular passage $F F' = a'$, and they therefore constitute a special kind of valve, the so-called ported-valve. But from what has preceded and what follows it seems best to regard *Gonzenbach's* valve as a *two-part* one. If we designate the distance $F F'$ between the two inner valve edges by a' and the width of the

passage in the partition by a_0, then the distance F E = F′ E′, formerly designated by s, may be obtained from the formula

$$s = a_0 + \frac{a' - a_0}{2} = \frac{a_0 + a'}{2}. \qquad (109)$$

Now if the two values a_0 and a' are given and the eccentricity r_0 and the negative angle of advance δ_0 are known, we get the diagram represented in Fig. 45*a* when we superimpose Fig. 42*a* and Fig. 43*a*. We need no further explanation to perceive that Fig. 45*a* will give us the following results.

Fig. 45.

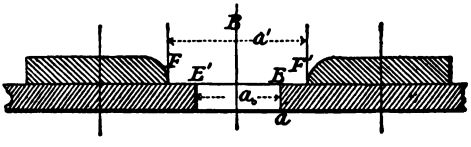
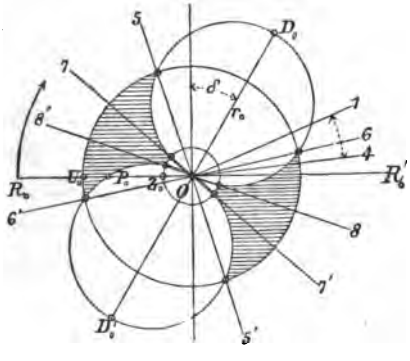


Fig. 45*a*.



Describe the two valve circles O D_0 and O D_0', also two other circles from O as a centre, and with

$$O\,U_0 = s = \frac{a_0 + a'}{2}$$

and

$$O\,Z_0 = s - a_0 = \frac{a' - a_0}{2}$$

as radii; the diagram is then complete. If the crank stands in position O R_0 at the first dead point and from there moves in the direction of the arrow, the distance $U_0 P_0$ will give the opening of the passage at the beginning of the stroke; at the crank position 8′ the passage is fully open, at position 7 the opening begins to close and at position 5 it is closed, expansion of the steam beginning here. The passage remains closed from position 5 to 6, at 6 it opens, and when the crank reaches the second dead point the opening is again equal to $U_0 P_0$. The same occurrences are repeated in the second semicircle. For fixed expansion, i. e. for the case in which all the separate parts here considered are invariable, the preceding discussion

has settled all questions connected with *Gonzenbach's* valve-gear; if, however, the problem were to examine an existing valve-gear, the question might still remain whether crank position 6 fell between positions 1 and 4, a question which was examined in connection with Fig. 41. It is worthy of notice that, in *Gonzenbach's* valve-gear, the expansion-valve eccentric may be set 180° from its assumed position without altering at all the distribution of the steam, for we see that the diagram is not altered thereby. The eccentric $O D_0$ in Fig. 18*a*, Pl. IV., can therefore be also brought into the dotted position $O D_0'$.

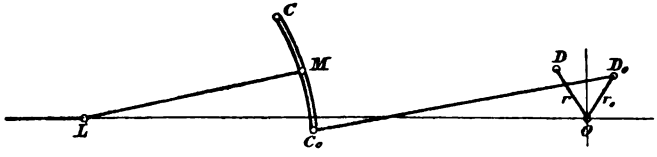
The question now remains how a *variable* expansion can be obtained by *Gonzenbach's* valve-gear; the problem here is to vary crank position 5 or the valve circle $O D_0$ or the circle described with $O U_0$ as a radius, and this variation must be effected while the engine is running if the arrangement is to be a practically useful one.

The variations above mentioned involve only three quantities, r_0, δ_0, and s; if any one or several of these be varied, the ratio of expansion will vary. As the distance $O U_0 = s$ (Fig. 45*a*) depends on the width a_0 of the passage and on the distance a' between the inner edges of the opening in the valve, we can vary one or both of these quantities; we see from this that a whole series of different arrangements may be devised and that our simple diagram indicates how new valve-gears may be invented. But all such efforts would be useless in connection with *Gonzenbach's* valve-gear, for this gear is, under all circumstances, unsuited for variable expansion, the variation being confined between too narrow limits. This is easily seen. Every shifting of crank position 5, whether effected by varying r_0, δ_0, a_0 or a', corresponds to an equal shifting of crank position 6. Now in discussing Fig. 41 it was shown to be absolutely necessary that position 6 should fall between positions 1 and 4 prescribed by the main valve, and as the angle formed by these two positions is always small, the shifting of position 5 for the purpose of producing variable expansion must always be confined within very narrow limits.

Consequently *Gonzenbach's* valve-gear cannot be recommended when it is desirable to produce variable expansion, and we find that it

has been rarely employed. Of the different methods of varying the expansion mentioned above, only one was employed by *Gonzenbach*, and that consisted in varying the eccentricity r_0 as follows.

FIG. 46.



The shaft O carries the eccentric O D $= r$ for the main valve and the eccentric O D_0 for the expansion-valve; the eccentric-rod $D_0 C_0$ of the latter is jointed at C_0 to a link C C_0 which swings about the fixed pin C; the radius-rod L M, which drives the stem L of the expansion-valve, can be clamped to any part of the link.

If the crank is moved from the dead point through the angle ω, the end C_0 of the link will have moved from its middle position the distance $r_0 \sin(\omega - \delta_0)$, and the point M of the link, which lies at the distance u from the fulcrum C, will have moved the distance

$$\xi_0 = \frac{u}{c} r_0 \sin(\omega - \delta_0) \tag{110}$$

from its middle position, c representing the length C C_0 of the link. The above expression at once gives the movement of the expansion-valve. The diagram Fig. 45*a* is also applicable, but in this case the diameter of the valve circle must be equal to $\frac{u}{c} r_0$, i.e. it must vary with the position of the radius-rod L M relatively to the link Fig. 46; every position of M has a special valve circle and every circle has a particular position for the crank directions 5 and 6. An example will illustrate more clearly what has been said.

In a valve-gear arranged as in Fig. 18*a*, Pl. IV., we have for the main valve the eccentricity $r = 0{\cdot}0675^m$ (2·66″), the angle of advance $\delta = 18°$, and the outside lap $e = 0{\cdot}016^m$ (0·63″).

On the other hand, for the expansion-valve the eccentricity $r_0 = 0{\cdot}0675^m$ (2·66″), the negative angle of advance $\delta_0 = 18°$, the

width of opening in expansion-valve $a' = 0 \cdot 0325^m$ ($1 \cdot 28''$), and the width of the passage in the partition $a_0 = 0 \cdot 025^m$ ($0 \cdot 98''$).

We are to ascertain whether these assumptions are allowable and what ratio of expansion these data will give.

Lay off from the vertical O Y, Fig. 19, Pl. IV., the angle of advance $Y O D = \delta$, make $O D = r$ and on it as a diameter describe the valve circle; moreover, with the outside lap $O V = e$ describe the lap circle; intersections V and V_1 then give the crank positions 1 and 4. At the former position the main valve closes the port for admission, at the latter the valve reopens the port, the crank turning in the direction of the arrow.

Also lay off from the vertical O Y the negative angle of advance $Y O D_0 = \delta_0$, make $O D_0 = r_0$ and on it describe the valve circle for the expansion-valve. Now determine from equation (109) the distance $F E = F' E' = s$ (Fig. 45), then we have

$$s = \frac{a_0 + a'}{2} = 0 \cdot 02875^m \ (1 \cdot 13'').$$

With this distance as a radius and O as a centre, describe a circle, the intersections U_1 and U give the two crank positions 5 and 6; at the former *expansion begins;* at the latter the passage reopens and we see that the arrangement is practicable, for position 6 falls between 1 and 4, as required by the conditions considered in connection with Fig. 41. If in addition we describe a circle from O as a centre and $O R_0 = 0 \cdot 050^m$ ($1 \cdot 97''$) as a radius, this will represent the crank-pin circle, and the parallel L_0 L to R_0 R will represent the piston stroke whose length is $0 \cdot 100$ ($3 \cdot 94''$).

If we drop from the crank-pin corresponding to position 5 a perpendicular R, L, on to the direction of the stroke, L_1 will represent the piston position at the beginning of expansion and the length L_0 L_1 expressed in decimetres gives the ratio of expansion; the measurement of the figure gives for this value $0 \cdot 135$.

Let us extend the consideration of this problem by assuming that the ratio of expansion $0 \cdot 200$ is to be produced without altering the eccentricity r_0 and the angle δ_0; for this purpose make $L_0 L^2 = 0 \cdot 020^m$ ($0 \cdot 79''$), drop a perpendicular from L_2 and prolong it till it intersects the crank-pin circle at R_2 and draw the crank position

R_2; then the distance O U_2 gives the value of s corresponding to the given ratio of expansion. By measurement we obtain from Fig. 19, Pl. IV., the value O $U_2 = s = 0 \cdot 0385^m$ (1·52″). If the width a_0 of the passage is to remain constant, we can alter the width a' of the opening in the expansion-valve so that equation (109) will be satisfied; then for the given ratio of expansion we find the width $a' = 0 \cdot 052^m$ (2·05″).

To give another example, let us assume that the valve-gear is to be arranged for variable expansion and that the expansion-valve is to be moved by a link as in Fig. 46; the construction given in Fig. 19, Pl. IV., is applicable, and the ratio of expansion 0·135 found above corresponds to the case in which the radius-rod L M is lowered as much as possible, the block M falls to C_0 and $u = c$ in equation (110). Now if the radius-rod is lifted, the diameter O $D_0' = \frac{u}{c} r_0$ of the expansion-valve circle becomes smaller and the points of intersection U_1 and U approach nearer to U″ and U′, the centres of the valve circles, corresponding to the different values of u, all lying on the line O D_0. There is a minimum value of u below which we must not go if we do not wish to have the steam enter the cylinder twice during a stroke of the piston; the dotted circle in the figure with diameter O D_0' corresponds to this value of u, for here the crank position 6 just coincides with position 1; as this determines the intersection U′ and as the circle in question passes through O and its centre C′ lies on the line O D_0, it is easy to find its diameter O D_0'; if the diameter were taken smaller the intersection U′ and consequently the crank position 6′ would fall *in front* of position 1, and that is not allowable. Measurement of the distance O D_0' in Fig. 19, Pl. IV., gives $0 \cdot 0365^m$ (1·44″), consequently

$$\frac{u}{c} r_0 = 0 \cdot 0365^m \ (1 \cdot 44'');$$

from this follows, since $r_0 = 0 \cdot 0675^m$ (2·66″),

$$\frac{u}{c} = 0 \cdot 541,$$

this will be the limit of the ratio C M : C C_0 (Fig. 46), and the slide-block M must not be moved nearer to C. Moreover, for this limiting

valve the intersection U″ (Fig. 19, Pl. IV.) gives the crank position O R_3 and the piston position L_3 at the beginning of the expansion; the distance L_0 L_3 gives the ratio of expansion 0·330. A greater cut-off is not possible unless the radius-rod is lifted so far that the block M comes close to the fulcrum C of the link; the expansion-valve is then entirely out of action and the main valve works alone.

In the present valve-gear the cut-off or ratio of expansion can only vary between the values 0·135 and 0·330, and these limits lie very close together. It is readily shown that varying the assumed dimensions will but slightly alter the result, and that in practice instead of *Gonzenbach's* valve-gear it is better to employ others which allow of greater variation of the expansion.

CHAPTER II.

Georges' Cut-off Gear.

Georges' valve-gear is represented in section in Fig. 20, Pl. IV.;* the middle portion S of the main valve has exactly the same form as the main valves hitherto examined. The valve is here extended on both sides, the additions S_1 S_1 forming two steam-ways *a a*.

On this valve rests the plate *c c* containing two rectangular openings, and on this plate is a second one *d d*, which is rigidly attached to *c c* and at the same time separates the valve-chest into an upper and lower chamber.

The upper plate *d d* also contains two rectangular passages a_0 a_0, which may be enlarged and contracted at pleasure by moving the two parts *n n* of the plate either toward or away from the centre; an arbor *l* passes through the upper cover of the valve-chest, and the inner end of the arbor carries a small toothed wheel *p*, which gears into two racks, one of which is fastened to one plate and the other

* *Armengaud*, 'Publication industrielle,' Tome ix.; 'Civilingenieur,' Bd. ii., 1856, S. 200.

to the other plate (see plan given in Fig. 20*a*). By turning the arbor *l*, the plates recede from or approach each other, thus contracting or enlarging the passages a_0 a_0 at will, and effecting the variable expansion. In order that the position of the plates and, consequently, the degree of expansion may be indicated on the outside, the arbor *l* has a disk *q*, which is graduated at the circumference; the index *c* fastened to the valve-chest then indicates the degree of expansion.

The expansion-valve as well as the main valve is here represented in the *middle position;* the former consists of a frame whose cross-plates S_0 S_0 in a certain sense constitute two separate but rigidly connected expansion-valves which are driven by the same eccentric.

Theoretically, *Georges'* valve-gear fulfils all the conditions of a perfect expansion-gear, for, when properly arranged, *all* degrees of expansion from zero to full stroke can be produced; it is only in constructive features that it is excelled by other valve-gears, hereafter to be described. This valve-gear avoids all the defects of *Gonzenbach's* gear because the main valve possesses two separate steam-ways *a a*, each of which receives steam from above; each of the two expansion-valves S_0 S_0 controls the supply for one of the steam-ways. The left-hand expansion-valve S_0 only regulates the cut-off for the left steam passages o_1 of the cylinder, and the right-hand valve S_0 regulates the cut-off for the cylinder passage o_2 on the right.

Gonzenbach's gear is quite different; there the steam admitted to the lower steam-chest by the expansion-valve, spreads itself to *both* sides of the main valve, and consequently the expansion-valve must shut and reopen the passage in the partition at the right moment for *both* cylinder passages o_1 and o_2. From the discussion connected with Fig. 45 and Fig. 45*a* we obtained the rule that after the passage in the partition has been closed by the expansion-valve at crank position 5, it must be reopened between positions 1 and 4; crank position 6 must lie *behind* position 1 and *in front of* position 4, in order that live steam may be admitted just before the beginning of a new stroke. This causes the ratio of expansion to be confined within such narrow limits in *Gonzenbach's* valve-gear.

The circumstances are much more favourable in *Georges'* valve-gear. Let us consider the left half of Fig. 20 (Pl. IV.), we may then easily deduce the following.

If the steam-piston moves toward the crank-shaft (from left to right in the figure), the main valve will be shifted to the right and the passage o_1 in the cylinder will be opened; in the first portion of the stroke, the expansion-valve S_0 also opens the passage a_0; at a certain piston and crank position (5) the expansion-valve closes the passage and expansion begins; later on (at 1) the main valve S closes the steam passage o_1.

But as regards the reopening of the passage a_0 by the expansion-valve, we see that it need not take place till the piston is *on the return stroke* and is near the *left* end of the stroke, i. e. till the crank has described nearly the whole of the *second* semicircle.

Consequently we obtain (Fig. 45*a*) incomparably greater room for crank position 6 than in *Gonzenbach's* arrangement. In the latter we have the condition that position 6 must fall between positions 1 and 4, but in *Georges'* gear we have only the condition that position 6 shall fall behind position 1; position 4 for the forward stroke need not be taken into account, but instead, we take into account position 4′ (prolongation of 4 beyond O) in the return stroke. In *Gonzenbach's* valve-gear the expansion-valve keeps the passage a_0 (Fig. 45*a*) closed while the crank is passing from 5 to 6, but in *Georges'* gear the passage is closed while the crank (turning in the direction of the arrow) traverses the long distance from position 1 to position 6′ in the second semicircle.

A closer examination of all double valve-gears, in which the motion of *both* valves is caused by eccentrics, shows that all gears can be divided into two groups whose difference consists in what has just been mentioned. Only *Georges'* gear and those which can be reduced to it, can be regarded as gears in which every degree of expansion can be obtained. It also readily follows from what has been said that in such gears the negative angle of advance δ_0 (Fig. 45*a*) has much greater freedom of location; it can be chosen much greater because the intersection of crank position 6 can lie anywhere in the second semicircle. *Georges'* assumed the angle

$\delta_0 = 90°$, and this value is particularly appropriate for the case in which the engine is to work with the same perfection, as regards variation of expansion, whether it is running forward or backward. As regards a special investigation of the action of *Georges'* gear by the aid of the diagram, we can say that in reality this matter was settled when the arrangements represented in Figs. 42, 43, and 44 were discussed. It will therefore suffice to show in a special case how we may determine the degree of expansion from the position of the plates *n n* (Fig. 20, Pl. IV.) and *vice versâ.*

Let O X (Fig. 20*b*, Pl. IV.) represent the direction of the valve-stroke and O Y the vertical through the centre of the shaft, and let O D represent the eccentricity of the main-valve eccentric and D O Y $= \delta$ the angle of advance; on the other hand, let the eccentricity O $D_0 = r_0$ of the expansion-valve eccentric for the negative angle of advance δ_0 coincide in direction with O X, for, like *Georges*, we will assume the negative angle of advance $\delta_0 = 90°$.

Now let the eccentricity of the main valve be $r = 0 \cdot 040^m$ $(1 \cdot 57'')$, the angle of advance $\delta = 15°$, and the outside lap $e = 0 \cdot 0085^m$ $(0 \cdot 33'')$; the intersection V of the lap circle with the valve circle O D (Fig. 21, Pl. IV.) gives the crank position 1 at which the main valve cuts off steam. Moreover, let O $D_0 = r = 0 \cdot 040^m$ $(1 \cdot 57'')$ be the eccentricity of the expansion eccentric, and on this as a diameter describe the valve circle.

For any position of the plates *n* (Fig. 20, Pl. IV.), the distance between the edges F and E will always be measured when the expansion-valve is in the *middle position* (see also Fig. 42); we will again designate this distance F E by *s* and will describe in Fig. 21 a circle from O as a centre and with O U $= s$ as a radius; the two intersections U and U_0 with the valve circle, when connected with O by a straight line, give the desired crank positions. At position 5 the expansion-valve cuts off the steam and the expansion begins, at position 6 the passage reopens; usually it is only the first crank position that is of interest.

It is now easy to understand the following remarks.

If we draw the plates *n* (Fig. 20) together till the distance F E $= s$ is identical with O U_1 in Fig. 21, the expansion and main valves

will cut off *simultaneously*; if the plates were drawn still closer, the upper valve would cut off *later* than the lower one, and finally if the distance s were taken greater than $O D_0$ (Fig. 21), the expansion-valve would not cut off at all, for it would be out of action because it would allow steam to flow through continually. Of course the greatest distance $F E = s$ will, in practice, be always taken smaller than $O U_1$ (Fig. 21).

Now if the plates n be gradually separated, the distance $F E = s$ will become smaller, the intersection U (Fig. 21) will travel along the valve circle $O D_0$ and continually approach O, the steam being cut off earlier and the ratio of expansion becoming less.

If the plates are separated till E comes under F (Fig. 20), we shall have $O U = s = O$ and the ratio of expansion 1 : 2. If the plates are separated still further, we shall have $F E = s$ negative and shall have the case discussed in Fig. 44. Our diagram remains unchanged. For example, if the edge E (Fig. 20) were at the left of F by the distance $O U = O U_0 = s$ (Fig. 21), then the crank directions O U and $O U_0$ would exchange their significations; if we prolong these directions beyond O, expansion begins at position 5′ and the passage reopens at 6′. If the plates n could be separated so far that the negative value $F E = s$ became identical with $O D_0 = r_0$ (Fig. 21), the expansion-valve would cut off steam at the beginning of the stroke, but there is no practical necessity for so doing.

Now let us settle the opposite question and assume that in the present valve-gear the ratio of expansion is 1 : 10, and for this determine the required position of the plates. We then proceed as follows:

From O (Fig. 21) as a centre and $O R = 0 \cdot 05^m$ ($1 \cdot 97''$) as a radius, describe the crank-pin circle and draw the line $L L_0$ parallel and equal to the distance $R R_0$; make the distance $L L_1 = 0 \cdot 01^m$ ($0 \cdot 39''$), through L_1 draw a vertical to R_1 on the crank-pin circle and prolong the crank position $O R_1$ beyond O to its intersection U′ with the valve circle $O D_0$. The distance $O U' = 0 \cdot 032^m$ ($1 \cdot 26''$) represents the distance that the edge E (Fig. 20) of the plate n must be to the left of the edge F of the expansion-valve, in order that the required ratio of expansion 1 : 10 may obtain.

This settles all questions connected with a given valve of the *Georges* type; we might now add rules for designing a new valve of this kind, but we will not enter into this question because, with the aid already given, the thoughtful designer can have no difficulty. We simply mention that in practice the upper and lower limits of expansion are always given. Then if the eccentricity $O D_0 = r_0$ (Fig. 21, Pl. IV.) has been chosen, we know the greatest positive and the greatest negative value of the quantity $O U = s$; the sum of the two gives the *total* shifting to be communicated to the plates n (Fig. 20), and from this we can easily determine the length to be given to the plate n and also the width a_0 of the passage. With equal ease we can determine the required length of the plates $S_0 S_0$ of the expansion-valve itself, a matter which has already been discussed in connection with Fig. 44.

It is worthy of mention that with Fig. 20 and the diagram in Fig. 21, Pl. IV., other valve-gear arrangements can be readily devised. Instead of contracting and enlarging the passages $a_0 a_0$ by adjusting the plates n n, i. e. *by varying the value of* s, we could, for example, fasten the plates n n and make the valve-plates $S_0 S_0$ movable; the diminution of s would then occur when the valve-plates were *drawn together*; or we could bring the plates $S_0 S_0$ together at the middle so that the *outer* edges would regulate the admission, the increase of expansion being effected by separating the plates; in this case the eccentricity $O D_0 = r_0$ (Fig. 20*b*) would have to be set 180° in advance of the position before assumed. It is also easy to see that the expansion can be varied by varying the amount and position of the eccentricity $O D_0 = r_0$. Valve-gears of the kind just mentioned have not thus far been executed.

SECTION II.

EXPANSION-VALVE ON THE BACK OF THE MAIN VALVE.

CHAPTER I.

BRÉVAL'S CUT-OFF GEAR.

IN *Bréval's* gear the two valves are in the same valve-chest K (Fig. 22, Pl. V.). The main valve S has here but little outside lap, and is, to a certain extent, surrounded by a case in which the steam-way $a\,a$ conducts the steam to the ends of the valve whenever the opening a' of the expansion-valve S_0 stands over the opening a_0 in the lower valve. If we imagine that the lower valve is fixed and that only the upper one moves, the admission and cutting off of the steam will take place as in *Gonzenbach's* valve-gear (Fig. 18, Pl. IV.); but as the lower valve does move, it is evidently the relative motion of the upper valve to the lower one that is required. We must first determine this motion, for it is needed for the discussion of all the valve-gears examined in this Section.

The two eccentrics which communicate motion to the two valves are fastened to the shaft as shown in Fig. 22*a*, Pl. V.; r is the eccentricity of the main eccentric, $Y\,O\,D = \delta$ its angle of advance, r_0 the eccentricity of the expansion eccentric, and $Y\,O\,D_0 = \delta_0$ its angle of advance. In determining the relative motion of the expansion-valve with reference to the main valve, we may at first entirely disregard the form of the valves. Let us suppose that on the upper surface A A (Fig. 47*a*) there slides back and forth a plate B driven by an eccentric $O\,D = r$ having the angle of advance δ (Fig. 48); suppose that on this plate B there slides a *second* plate B_0 driven by the eccentric $O\,D_0 = r_0$ having the angle of advance $Y\,O\,D_0 = \delta_0$;

the valve circles described on the eccentricities O D and O D_0 will then separately give the absolute motion of their respective plates. In Fig. 47*a* *both* plates are shown in their middle position, which is only possible when they are detached from their eccentrics.

FIG. 47*a*.

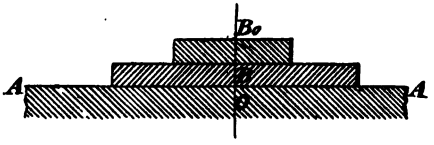


FIG. 47*b*.

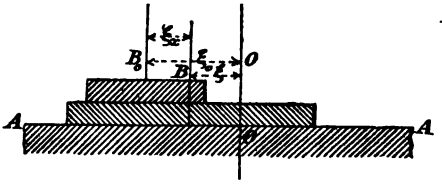
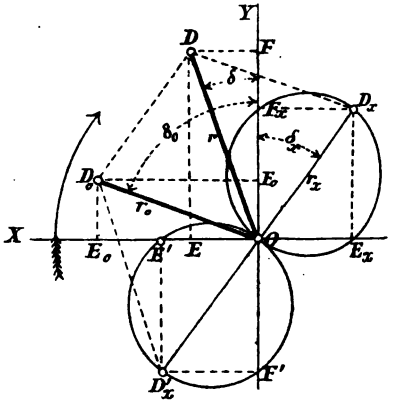


FIG. 48.



Now if we suppose them connected with their eccentrics and the latter to be turned with the crank through the angle ω, then for this crank angle we have (Fig. 47*b*)

$$\mathrm{O\,B} = \xi = r \sin(\delta + \omega)$$

for the movement of the lower plate from its middle position, and at the same instant the movement of the upper plate is

$$\mathrm{O\,B_0} = \xi_0 = r_0 \sin(\delta_0 + \omega).$$

The relative distance or movement of the upper valve with reference to the lower valve is $\mathrm{B\,B_0}$, as a glance at Fig. 47*b* shows; if we designate this relative movement by ξ_x, we have

$$\xi_x = \xi_0 - \xi,$$

or, according to the preceding equations,

$$\xi_x = r_0 \sin(\delta_0 + \omega) - r \sin(\delta + \omega),$$

or after simple reduction,

$$\xi_x = (r_0 \sin \delta_0 - r \sin \delta) \cos \omega - (r \cos \delta - r_0 \cos \delta_0) \sin \omega. \quad (111)$$

This equation gives for any crank angle ω the *relative motion of the upper plate with reference to the lower*, or of the expansion-valve with reference to the main valve; we do not therefore alter the relative motion if we regard *the lower plate as stationary and the upper one moved according to the law given by the above equation.* But this law is very simple, for the equation is again the polar equation of a circle, the coefficients of $\cos \omega$ and $\sin \omega$ being constants.

If we lay off on O X the value

$$O E' = O E_x = r_0 \sin \delta_0 - r \sin \delta = A$$

(Fig. 48), and on O Y the value

$$O F' = O F_x = r \cos \delta - r_0 \cos \delta_0 = B,$$

and describe circles through the two sets of points O F′ E′ and O F_x E_x, we shall get valve circles whose radius vectors give the movement of the upper valve relatively to the lower valve for any crank position.

The relative motion is therefore exactly as if the lower valve were stationary and the upper were moved by an eccentric whose eccentricity $O D_x' = r_x$ is fastened to the shaft with the angle of advance $D_x' O F' = Y O X = \delta_x$.

We shall call these two valve circles, which respectively represent the positive and negative movements of the upper valve relatively to the lower, the relative valve circles, and the value $O D_x = r_x$ we shall call the relative eccentricity; usually it will be necessary to describe only one valve circle $O D_x$.

Moreover, the relative valve circle can easily be found without any calculation whatever. If (Fig. 48) we let fall from the ends D and D_0 of the eccentricities $O D = r$ and $O D_0 = r_0$, perpendiculars on the axes O X and O Y, we have

$$D_0 F_0 = O E_0 = r_0 \sin \delta_0$$

and

$$D F = O E = r \sin \delta,$$

also

$$D_0 E_0 = O F_0 = r_0 \cos \delta_0$$

and

$$D E = O F = r \cos \delta;$$

accordingly,

$$E E_0 = r_0 \sin \delta_0 - r \sin \delta$$

and

$$F F_0 = r \cos \delta - r_0 \cos \delta_0.$$

It now follows from the above, that

$$O E_x = E E_0$$

and

$$O F_x = F F_0.$$

The diagonals $O D_x$ and $O D_x'$ of the two rectangles constructed on these distances are then the required diameters of the relative valve circles; they represent the relative eccentricity r_x and their position gives the negative angle of advance $Y O D_x = \delta_x$, but we see that the distances $O D_x$ and $O D_x'$ are simply the *second sides of two parallelograms constructed with the given eccentricities* $O D = r$ and $O D_0 = r_0$; either $O D$ or $O D_0$ may be taken as the diagonal, in the first case $O D_x$ will be the second side, in the other case $O D_x'$, but these sides are equal and opposite.

We now perceive the simple connection between the two principal kinds of double valve-gears discussed in this and the preceding Section.

If we compare Fig. 48 with Fig. 45*a*, we see that in both cases we have the same diagram; the difference is simply that the eccentricity r_0 must be fastened to the shaft with a negative angle of advance when the expansion-valve slides on a stationary plate, as in *Gonzenbach's* and *Georges'* gear, and that in place of $O D_0 = r_0$ (Fig. 45*a*) we have the relative eccentricity $O D_x = r_x$ (Fig. 48) whenever the upper valve moves on the lower one. Therefore if we have a *Bréval's* valve-gear, in which the relative eccentricity $O D_x$ (Fig. 22*a*, Pl. V.) is equal in magnitude and position to the eccentricity $O D_0$ of the expansion eccentric (Fig. 18*a*, Pl. IV.) belonging to *Gonzenbach's* valve-gear, and if the widths of the openings a_0 and a' (Fig. 22, Pl. V., and Fig. 18, Pl. IV.) are equal, then the two valve-gears will have exactly the same distribution of steam, provided the main valves have the same laps and are both moved by eccentrics having the same eccentricity and the same angle of advance. We might therefore repeat for *Bréval's* gear, all that was said in con-

nection with Fig. 45 and Fig. 45*a*. It follows that *Bréval's* valve-gear possesses in the same degree, all the imperfections mentioned in connection with *Gonzenbach's* gear, and is, consequently, as little to be recommended. Moreover, the example represented by Fig. 19, Pl. IV., can also be transferred directly to *Bréval's* gear; if, for the latter gear, the distance $O D_0$ in this figure is to represent the *relative* eccentricity, we must take the eccentricity of the expansion-valve $D D_0 = O D_0' = 0 \cdot 042^m$ ($1 \cdot 65''$), and must fasten the eccentric with an angle of advance of 90°. These indications must suffice, for *Bréval's* gear was only mentioned as a companion piece to *Gonzenbach's* gear, for the sake of completeness; in like manner the following valve-gears will show that they are companion pieces to *Georges'* much more perfect valve-gear.

CHAPTER II.

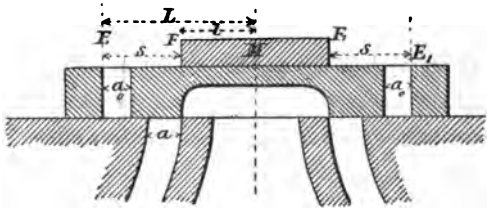
"Polonceau's" (or Borsig's) Cut-off Gear.

Different arrangements of this gear are much used, the difference in the arrangements consisting in the manner in which the expansion is varied. A few of the most important cases will be discussed in what follows, but these will be preceded by an examination of the simplest case, in which there is fixed expansion.

Fig. 23, Pl. V., shows the arrangement of the valves of this gear, both valves being regarded as detached from their eccentrics and at the middle position. The ordinary valve S is provided at each end with additions which create the two steam-ways a_0 a_0; on this valve slides the expansion-valve S_0, which consists of a simple plate and passes over the steam-ways alternately. The eccentrics are fastened to the driving shaft as in *Bréval's* gear, Fig. 22*a*, Pl. V. The valve arrangement is again given in Fig. 49, the length of the expansion-valve is $F F_1 = 2l$ and its outer edges are at the distance $F E = F_1 E_1 = s$ from the outer edges of the steam-ways. Here, as in *Georges'* valve-gear, it suffices to examine the distribution of steam on one side, and the main problem is to find the crank positions at

which the steam-way a_0 is closed and reopened by the expansion-valve (positions 5 and 6 according to our former notation). In a given valve-gear, moreover, we must see whether the condition is fulfilled, that position 6 shall fall behind position 1 (see p. 182).

Fig. 49.



For example, if, in an existing valve-gear, there were given the eccentricity $r = 0 \cdot 040^m$ ($1 \cdot 57''$) of the main eccentric, the angle of advance $\delta = 15°$, the eccentricity $r_0 = 0 \cdot 045^m$ ($1 \cdot 77''$) of the expansion eccentric, the angle of advance $\delta_0 = 85°$, the outside lap $e = 0 \cdot 0085^m$ ($0 \cdot 33''$) of the lower valve, the width of the steam-way $a_0 = 0 \cdot 020^m$ ($0 \cdot 79''$), and the distance $FE = s = 0 \cdot 030^m$ ($1 \cdot 18''$), then we could determine from what has already been said, the distribution of the steam as follows. Construct the parallelogram (Fig. 24, Pl. V.) on the two eccentricities $OD = r$ and $OD_0 = r_0$; its second side $OD_x = r_x$ is then the relative eccentricity and the circle described on it is the relative valve circle which represents the motion of the upper valve relatively to the lower one. Now if we describe a second circle from O as a centre and $OU_0 = s$ as a radius, we get the two intersections U_1 and U_2; crank position 5 passes through the former and corresponds to the instant at which cut-off or the beginning of expansion takes place; crank position 6 passes through the other point of intersection U_2 and corresponds to the reopening of the steam-way a_0. Adding the diagram for the main valve, crank position 1 corresponds to the instant at which the main valve closes the steam passage in the cylinder. As position 6 is here behind position 1, the valve-gear is a practicable one; moreover, $0 \cdot 435$ represents the ratio of expansion for crank position 5, as may readily be proved by constructing the corresponding piston position. Finally, the valve-link $2l$ must satisfy the condition that the following or

back edge of the valve shall not stand over the steam-way when the valve is at the extreme end of its travel; but the greatest relative movement is r_x, hence we must, as Fig. 49 at once shows, have

$$2\,l + s - a_0 > r_x,$$

or the whole length of the plate must be

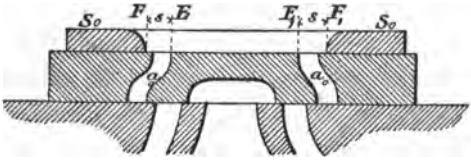
$$2\,l > r_x + a_0 - s,$$

a condition always easy to fulfil.

The consideration of *Polonceau's* valve-gear may be immediately followed by that of *Borsig*, an arrangement employed on numerous *Borsig* locomotives. This valve-gear is closely related to that of *Polonceau*, and their diagrams are identical as we shall see.

If in Fig. 49 we suppose the expansion-plate $F\,F_1$ to be divided in the middle and the two parts $S_0\,S_0$ to be arranged for the middle position, as shown in Fig. 50, we shall have *Borsig's* valve arrangement, the distances of the *inner* edges $F\,F_1$ of the plate from the *inner* edges E and E_1 of the steam-ways being $F\,E = F_1\,E_1 = s$. We

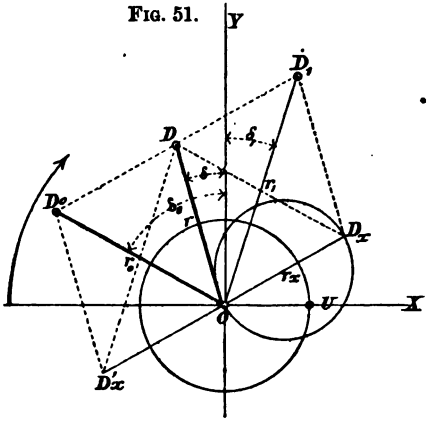
FIG. 50.



see that the expansion-valve is an open one like that of *Georges*, but the latter slides on the *stationary* cover of the lower steam-chest. It has been already mentioned in connection with *Georges'* gear, that the plates can be united at the centre, and the outer edges instead of the inner ones be the controlling ones. It was there stated that with this variable arrangement (which, so far as we know, has not been constructed), it is simply necessary to change the position of the expansion eccentric in *Georges'* valve-gear 180°. This is exactly what must take place in transforming *Polonceau's* gear into that of *Borsig*; but here it is the *relative eccentricity* which is shifted 180°, and this is effected by choosing another eccentricity for the expansion-valve and another angle of advance, the following considerations serving as points of departure. Let us assume that in *Polonceau's*

valve-gear we have laid off the eccentricities r and r_0 in the manner described above, namely, by constructing a parallelogram on O D as a diagonal, then O D_x will be the relative eccentricity r_x; let us also assume that for the same main valve we are to so construct *Borsig's* arrangement (Fig. 51) that with the same value of s, the same distribution of steam will be effected, as in *Polonceau's* gear; then the *relative* eccentricity O D_x must be shifted 180°, i. e. must occupy the position OD_x'. Now construct on O D_x' as a side and O D = r as a diagonal, a parallelogram OD_x' DD_1, then O D_1 = r_1 is *the amount and position of the eccentricity of the eccentric which must govern the expansion-valve in Borsig's arrangement.* In general, as in the present case, the eccentricity r_1 of *Borsig's* expansion eccentric is different from r_0 in *Polonceau's* gear, and besides, the former is fastened to the shaft with a negative angle of advance δ_1 and the latter with the angle of advance δ_0. If we describe on O D_x as a diameter, the relative valve circle, and from O as a centre and O U = s as a radius a second circle, and suppose the diagram crank to turn in the direction of the arrow, the diagram in Fig. 51 will apply to both *Polonceau's* and *Borsig's* arrangements, consequently there is nothing new to be said with reference to the latter. We may remark that in *Borsig's* locomotives the main valve is driven by *Stephenson's* link-motion with crossed rods, and the expansion-valve by the back eccentric through the medium of a double-armed lever. But when the locomotive is reversed *Borsig's* expansion-gear will not work, which is not the case with *Polonceau's* gear as applied to locomotives and as described below.*

FIG. 51.



* *Borsig's* locomotive valve-gear has been discussed with the aid of our diagram, by *Lehmann*, in an article entitled "Die *Borsig'sch* Locomotivsteuerung," in the 'Organ für die Fortschritte des Eisenbahnwesens,' 1858, Bd. xiii., S. 241.

Let us return to *Polonceau's* gear, and discuss the important problem of the means to be employed to produce variable expansion.

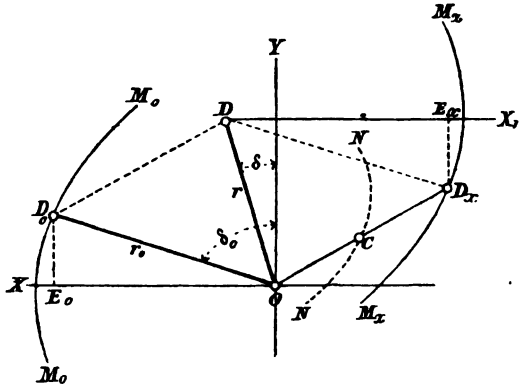
We may vary the expansion by varying any one of three quantities, namely, the eccentricity r_0 of the expansion eccentric, its angle of advance δ_0, or the quantity F E = s (Fig. 49). The quantity s may be varied in either of two ways; one way is to make the width a_0 (Fig. 49) of the steam-ways variable (as in *Georges'* gear), but this way has not thus far been employed in practice on account of its constructive difficulties; the other way is to make the length F $F_1 = 2\,l$ of the plate variable. This can be accomplished by dividing the plate in the middle and arranging the two parts so that they can be set at any desired distance from each other; this arrangement is known as *Meyer's* gear and will be examined hereafter.

Let us now examine the method of varying the expansion by varying r_0 and δ_0, the quantity s being assumed as constant. This method was the one followed by *Polonceau*, and has recently been further developed by *Guinotte*. Here it is a question of *simultaneously* varying *both* quantities, and the law of this variation will depend on the mechanism which governs the motion of the expansion-valve; the case in which *only* the angle of advance δ_0 is variable, constitutes a particular case of the general one. The arrangements designed by *Polonceau* and *Guinotte*, when examined by the aid of our diagram, can be referred to the following general proposition.

If, as before, O D (Fig. 52) is the eccentricity r of the main eccentric and δ the angle of advance, and if we assume that these two quantities are invariable, but that the eccentricity O $D_0 = r_0$ and the angle δ_0 of the expansion eccentric are variable, then we can imagine a mechanism which will vary r_0 and δ_0, while the machine is running, in such a way that the end D_0 of the eccentricity r_0 will traverse the curve M_0 M_0. Every different position and magnitude of the eccentricity O D then corresponds to a particular position and value of the *relative* eccentricity O D_x; according to the above this is simply the second side of a parallelogram described on O D as a diagonal and O D_0 as a first side. The end D_x will therefore traverse a second curve M_x M_x whose course can easily be determined when the first curve M_0 M_0 is given.

If from D_0 we drop the perpendicular $D_0 E_0$ on to the horizontal O X, the distances $O E_0 = x$ and $E_0 D_0 = y$ can be regarded as co-ordinates of the point D_0 of the curve $M_0 M_0$, and we may suppose the curve to be given analytically by an equation expressing the difference between x and y.

FIG. 52.



If we now draw, through the end D of the fixed eccentricity O D of the main valve, the line $D X_1$ parallel to O X, and let fall from D_x the perpendicular $D_x E_x$ on $D X_1$, we may regard the distances $D E_x$ and $E_x D_x$ as co-ordinates of the fourth vertex D_x of the parallelogram. The figure at once shows that $D E_x = O E_0 = x$ and $D_x E_x = D_0 E_0 = y$, and from this follows the proposition that under all circumstances the curve $M_x M_x$, traversed by the end D_x of the *relative* eccentricity, is like the curve $M_0 M_0$ traversed by the end D_0 of the eccentricity O D_0 of the expansion eccentric. But the co-ordinates of the points of the second curve have signs which are the opposites of those of the first curve, and the origin of co-ordinates of the second is at D while that of the first is at O.

If from O we draw radius vectors $O D_x$ to different points of the curve $M_x M_x$, and bisect these vectors, connecting the points of bisection by a line, we shall obtain a new curve N N containing the centres C of the relative valve circles; this curve is similar to the given curve $M_0 M_0$. The curve N N is the one which will principally engage our attention, and we will call it the relative curve of centres; in the

systems thus far constructed (*Polonceau's* and *Guinotte's*) the curves are of very simple character, for the original curve $M_0 M_0$ is a straight line; the curves $M_x M_x$ and N N are therefore straight lines also, and are parallel to the line $M_0 M_0$.

Before investigating these valve-gears we will discuss one special case which occurs in stationary engines and differs from the above-mentioned gears in that the curves are not straight lines but circles.

Thus we can vary the expansion by turning the expansion eccentric on the shaft; in this case the eccentricity $O D_0 = r_0$ is invariable, and only the angle of advance δ_0 is variable. The curve $M_0 M_0$ in Fig. 52 then becomes a circular arc described from the shaft centre O with a radius $O D_0 = r_0$. Moreover, we know from what has preceded that the curve $M_x M_x$, traversed by the end D_x of the relative eccentricity, is likewise a circle having the radius $D D_x = r_0$ described from D as a centre. The relative curve of centres is then, evidently, a circle whose radius is $\frac{1}{2} r_0$ and whose centre is at the middle of the distance O D. In Fig. 24, Pl. V., the example discussed on p. 204 is extended; the relative curve of centres N N is dotted. The eccentricity of the main eccentric is $r = 0 \cdot 040^m$ $(1 \cdot 57'')$, the angle of advance $\delta = 15°$, the corresponding quantities for the expansion eccentric are $r_0 = 0 \cdot 045^m$ $(1 \cdot 77'')$, and $Y O D_0 = \delta_0 = 85°$, and the distance $O U_0 = s$ is assumed equal to $0 \cdot 030^m$ $(1 \cdot 18'')$. From what was said on p. 204 we see that the distance $O D_x$ gives the diameter of the relative valve circle and that the intersections U_1 and U_2 represent the two crank positions 5 and 6, one of which corresponds to the beginning of expansion, and the other to the reopening of the steam-ways.

Now if the expansion eccentric $O D_0$ is shifted, say, so that the eccentricity passes from $O D_0$ to $O D_0'$, it is simply necessary to repeat the given construction; the diameter of the relative valve circle is then $O D_x'$, and the intersections of this circle with the circle described with $O U_0 = s$ as a radius gives new positions for the crank directions 5 and 6; at the same time the end of the relative eccentricity passes from D_x to D_x' and the centre of the relative valve circle passes from C to C'. In practice the problem is given in the inverse form, i. e. the position $O D_0'$ of the expansion eccentric is to be determined for a given ratio of expansion. For example, if in the present case the ratio of

expansion 4 : 1 is prescribed, we determine the corresponding crank position O R, find the intersection U and erect a perpendicular at the middle of the radius O U; this intersects the relative curve of centres at C′, which is therefore the centre of the required relative valve circle whose radius is C′ O; if we draw its diameter O D_x' and draw O D_0' parallel to the connecting line D D_x', the line O D_0' will represent the required position of the expansion eccentric and the angle Y O D_0' will be the required angle of advance δ_0 for the given ratio of expansion.

After what has been said above, the remarks just made need no further proof; neither is it necessary to discuss the method of determining the limits of expansion in the given arrangement, for the extreme positions of the relative valve circle can be readily found by shifting its centre C on the relative curve of centres N N.

We will now examine more closely the arrangement given to the valve-gear by *Polonceau* himself, to render it suitable for locomotives.

This valve-gear is shown in Fig. 25, Pl. V. The two eccentrics D and D_0 on the shaft O set in motion a *Gooch* link of the kind discussed in Section I., pp. 107 to 121. But the link is *double*, that is, it has two equal curved slots or guides. In one slot wec an adjust the block M of the rod M N which at N drives the stem of the main valve. In the other slot we have the block M_0 of the rod M_0 N_0 which drives the expansion-valve. The shifting of the blocks by lifting and lowering the radius-rods M N and M_0 N_0 is accomplished by the levers A B and A_0 B_0 whose ends B and B_0 are connected by rods with the bell-crank levers W W and W_0 W_0; the rods K and K_0 transmit the shifting motion to the radius-rods. By means of latches and the notches in the sector H H, the reversing levers are held fast in different positions.

It is easy to see that this valve-gear becomes like the *Gooch* gear when the two blocks M and M_0 are set at the same point of the link; for then the two valves move in *the same* manner and the expansion-valve has no influence because it is at rest relatively to the main valve; but when the blocks have *different* positions in the link, the valves move differently, and the problem of ascertaining how the expansion-valve moves relatively to the main valve, again presents itself.

When it is desirable to throw the expansion-valve temporarily

out of action, *Polonceau's* arrangement of the reversing levers is inconvenient, for the locomotive runner must use both hands to give the same motion to the two levers. This inconvenience is avoided in *Krauss'* arrangement, shown in Fig. 26, Pl. V. H H is again the stationary sector, but its notches receive only the latch on the reversing lever A B of the main valve. A second sector $H_0 H_0$ is *fastened* at P to the lever A B, the sector taking part in the motion of the lever. The notches of *this* sector receive the latch of the reversing lever $A_0 B_0$ of the expansion-valve. It is now easy to see how the levers are used. In the lever positions shown in Fig. 26, the valves are in action in nearly the same manner as in the arrangement shown in Fig. 25. Now if the runner wishes to throw the expansion-valve out of action, i. e. to use the gear as if it were a simple *Gooch* link-motion, he places the latch of the lever $A_0 B_0$ in the notch P of the second sector $H_0 H_0$. Then when the lever A B is shifted, the other lever is also moved. As regards the action of *Polonceau's* arrangement, we may refer to much that has been mentioned in connection with *Gooch's* gear. But under all circumstances we may assume that the radius-rod of the main valve is set for the extreme grade of expansion whenever the expansion-valve is to be used. Under this supposition we will first determine the distribution of steam for the main valve, i. e. we will determine its principal crank positions.

As in *Gooch's* gear, we will designate the eccentricities of the two eccentrics by r, their equal angles of advance by δ, the lengths of the two eccentric-rods C D and $C_0 D_0$ (Fig. 25, Pl. V.) by l, the half length of the link by c, and the distance of the block M from the dead point of the link by u, then the movement of the valve for any crank angle ω is

$$\xi = r\left[\sin\delta \pm \frac{c}{l}\cos\delta\right]\cos\omega \pm \frac{u\,r}{c}\left[\cos\delta \mp \frac{c}{l}\sin\delta\right]\sin\omega,$$

the upper sign referring to open and the lower one to crossed rods. Now in what follows we will assume open rods and that the radius-rod of the main valve is lifted so far that $u = c$, then the movement of this valve is given by the formula

$$\xi = r\left[\sin\delta + \frac{c}{l}\cos\delta\right]\cos\omega + r\left[\cos\delta - \frac{c}{l}\sin\delta\right]\sin\omega.$$

We may now find, as before, the co-ordinates of the centre C of the valve circle for the extreme grade of expansion (Fig. 27, Pl. V.), from

$$O\,B = \frac{r}{2}\left[\sin\delta + \frac{c}{l}\cos\delta\right]$$

$$B\,C_4 = \frac{r}{2}\left[\cos\delta - \frac{c}{l}\sin\delta\right].$$

Fig. 27 is drawn half size, and with the dimensions

$$r = 0{\cdot}066^{m}\ (2{\cdot}60'') \qquad \delta = 30^{\circ}$$
$$l = 1{\cdot}500^{m}\ (59{\cdot}06'') \qquad c = 0{\cdot}120^{m}\ (4{\cdot}72'').$$

The foregoing formulas give, for these dimensions, the quantities

$$O\,B = 0{\cdot}0188^{m}\ (0{\cdot}74'')$$

and

$$B\,C_4 = 0{\cdot}0272^{m}\ (1{\cdot}07'').$$

Moreover, if we assume the outside lap of the main valve to be $O\,V = e = 0{\cdot}03^{m}$ $(11{\cdot}81'')$, and, with this as radius, describe a circle from O as a centre, its intersections with the valve circle will give crank positions 1 and 4; at position 1 the main valve closes the admission-port in the cylinder, and at position 4 it reopens the corresponding port at the other end of the cylinder.

Now if the block M_0 belonging to the expansion-valve is at any other point of the link, the centre of the corresponding valve circle will still lie on the vertical $B\,C_4$, a property that was shown to belong to *Gooch's* gear. If four grades of expansion are assumed for the forward motion and as many for the reverse motion, the distance $B\,C_4$ must be divided into four parts, the points $C_1\,C_2\,C_3\,C_4$ giving the centres of the valve circles corresponding to the different grades. For the reverse motion the distance $B\,C_4$ is prolonged downwards, the points $-C_1$, $-C_2$, $-C_3$ and $-C_4$ likewise giving the centres of the valve circles for the respective grades of expansion during the reverse motion.

If we assume that the machine is running forward and that the link-block M of the main valve is at the extreme grade, then, from what has preceded, we know that the circle described on O D as a diameter (Fig. 27, Pl. V.) gives the law of motion of the main valve. Moreover, if the link-block M_0 of the expansion-valve stands, say, at the first grade of expansion, for the forward motion, C_1 will be the

centre of the valve circle which represents the law of this valve motion and O D_0 will be its diameter. If now we draw a parallelogram on O D as a diagonal, we get O D_x for the second side, and this represents the diameter of the relative valve circle whose centre O_1 can also be found by drawing through C_4 a parallel C_4 O_1 to C_1 O.

We see that the peculiarity of *Polonceau's* arrangement consists in the relative curve of centres being a straight line which *coincides with the axis of ordinates O Y.* If we now describe from O_1 as a centre, the relative valve circle, and from O as a centre with radius O U = *s* (see Fig. 23, Pl. V.) a circle, we shall get the important points of intersection U_1 and U_2. The crank direction O R_1 passing through U_1 corresponds to the beginning of the expansion, and the other through U_2 to the reopening of the steam-way. The latter crank direction hitherto designated as position 6, happens in the present example to coincide with position 1, at which the main valve closes its port; as position 6 cannot lie in front of position 1, the position of the link-block M_0 here assumed (first grade forward motion) corresponds to one limit of expansion of the present gear; a later cut-off is not permissible because steam would be admitted to the cylinder twice during the same stroke. If a later cut-off must be employed on the locomotive, the runner must place the radius-rod of the main valve and that of the expansion-valve together at some other than the last grade of expansion, i. e. he must allow the gear to act like *Gooch's* link-motion.

If we now suppose the radius-rod of the main valve to be placed at the last (4) grade of expansion, and if we successively place the link-block M_0 of the expansion-valve at the dead point of the link and at grades 1, 2, 3, 4, *back-gear,* then we determine the centres of the relative valve circles exactly as before, these centres all lying on the vertical O Y and occupying the positions O_0, $-O_1$ to $-O_4$ respectively, in Fig. 27, Pl. V. If we describe the corresponding circles which are only partly drawn in the figure, their intersections with the circle described with O U = *s* as a radius, will give the points U_1 to U_6 and these in turn the crank positions R_1 to R_6, each of which represents the beginning of the expansion for its particular case. In the present example the crank position O R_6 gives the other limit of expansion

corresponding to the shortest cut-off here attainable; the link-block M_0 of the expansion-valve then stands at the extreme grade back-gear. We could extend the limits of expansion by increasing the length $C C_0$ (Fig. 25, Pl. V.) in both directions, and thus permitting the link-block M_0 to be placed beyond the points C and C_0 of the eccentric-rods, but on the whole little would be gained.

That the present gear can be employed for the reverse motion as well as for the forward motion is at once evident; for this purpose the link-block M of the main valve is placed at the extreme grade back-gear, and the link-block M_0 at any grade forward gear. The diagram, Fig. 27, Pl. V., is then at once applicable to the reverse motion.

Finally, it deserves mention that *Polonceau's* gear, in addition to the disadvantage of a limited expansion, has another disadvantage, namely, that the loss of work due to the friction of the expansion-valve increases with the expansion. For if we designate the diameter $O D_x$ of the relative valve circle by r_x, the distance traversed by the upper valve on the lower one during one revolution of the shaft is given by $4 r_x$. The loss of work due to friction between the valves, is proportional to the distance traversed, and therefore *proportional to the value* r_x; Fig. 27, Pl. V., shows that this value increases with the expansion, hence the work lost by friction increases likewise. A similar remark might be made with regard to *Gonzenbach's* gear, in the special discussion on p. 192.

CHAPTER III.

Guinotte's Cut-off Gear.

At the Vienna Exposition certain new and very ingenious valve-gears attracted well-deserved attention, some of them being in the form of models, and some on locomotives. Before their appearance at the Exposition, the inventor *Guinotte* had made them known in a special article.* *Guinotte* gives a series of new arrangements, all of

* *Lucien Guinotte*, 'Étude générale sur la détente variable et spécialement sur son application aux machines d'extraction.' Mons, 1871.

which may be referred to the same principle; we will therefore confine ourselves to two of the new gears, referring the reader to the article itself for further details. Although *Guinotte* used our diagram extensively in his article, and says that he found it very useful in examining and representing the apparently complicated motions of his mechanisms, nevertheless as a continuation of the above discussion we will pursue, in what follows, a different method from that employed by *Guinotte*. In the first place, the valve motion in *Guinotte's* gears is only a particular case of the general one examined in connection with Fig. 52, p. 208; in the second case it seems desirable to examine the matter analytically in order to recognise the influence of the dimensions of the various parts of *Guinotte's* gear, on the distribution of the steam.

Guinotte uses *Polonceau's* expansion-valve (Fig. 23, Pl. V.) in his gear, and like him produces variation of expansion by simultaneously varying the eccentricity r_0 and the angle of advance δ_0 of the expansion eccentric; but the law according to which this variation takes place in his mechanism differs from that of *Polonceau*, and in a way that must be regarded as a decided improvement. Without going into the details of *Guinotte's* gears, we can examine their advantages and the principle underlying them.

In Fig. 53 as in Fig. 52, let $O\,D = r$ be the fixed eccentricity and $Y\,O\,D = \delta$ the angle of advance of the eccentric for the main valve, also let $O\,D_0 = r_0$ be a particular position of the *variable* eccentricity of the expansion eccentric; then the peculiarity of *Guinotte's* gear consists in the curve $M_0\,M_0$, traversed by the end D_0 of the eccentricity, being a *straight line* which is inclined to the axis of co-ordinates. If we construct the parallelogram and thus determine the position OD_x of the relative eccentricity, and then draw through D_x the line $M_x\,M_x$ parallel to $M_0\,M_0$, we shall have the path traversed by the end D_x of the *relative* eccentricity. Finally, if we bisect the distance $O\,D_x$ at C and through this point draw a parallel N N, this line, dotted in the figure, will be the relative curve of centres along which the centre of the relative valve circle moves, when the eccentricity of the expansion-valve varies. The principal object of the investigations is the determination of this relative curve of centres and

the essential feature of *Guinotte's* gears is that the line N N has the position shown in the figure and is *inclined* to the axis O X.

We can already see the relationship between *Guinotte's* and *Polonceau's* gears; in the latter the aforesaid curves are likewise

Fig. 53.

straight lines (see Fig. 27, Pl. V.), but they are perpendicular to the axis O X, and at the same time the curve M_x M_x as well as the relative curve of centres N N coincides with the vertical O Y. But it is just the latter circumstance which restricts expansion in *Polonceau's* gear. In the case represented in Fig. 27, Pl. V., the crank directions O R_1 and O R_6 give the limits of the expansion; crank position O R_1 corresponds to the least expansion or latest cut-off (when an expansion-valve is used) and position O R_6 to the greatest expansion or earliest cut-off. In a perfect cut-off gear it should be possible to vary the expansion from zero up to the degree attainable by the main valve alone, in other words, the intersection U_1 (Fig. 27, Pl. V.) should be brought to the position U_2 at which the main valve shuts the cylinder-port, and the intersection U_6

should be transferred to U_0 in order that the expansion-valve may cut off at the very beginning of the stroke; this last has the advantage that the expansion-valve alone could suppress the admission of steam to the cylinder. In *Polonceau's* gear it is impossible to fulfil these conditions, as a glance at Fig. 27, Pl. V., shows, for the centre of the relative valve circle travels on the vertical, and it would have to be at an infinite distance to allow the intersection U_6 to fall on U_0.

But matters are very different in *Guinotte's* gears where, as we will show, the relative line of centres N N (Fig. 53) is a straight line inclined to the axis O X. If we describe a circle from O as a centre and with $O U_0 = s$ as a radius (the meaning of s is seen in Fig. 26, Pl. V.) and determine crank position 1 corresponding to the beginning of the expansion effected by the main valve, it will only be necessary to effect each grade of expansion from the dead point $O R_0$ to crank position 1, i. e. it will only be necessary to let the intersection of the relative valve circle with the circle described with s as a radius, traverse the whole arc from U_0 to U_2.

If at the middle of the distance $O U_2$ (Fig. 53) we erect a perpendicular until it intersects the line N N at C_1 and erect another at the middle of $O U_0$ prolonging it till it intersects the line of centres N N at C_2, these two intersections will give the extreme positions of the centres of the relative valve circles; if we describe from C_1 as a centre and $C_1 O$ as a radius, one, and from C_2 as a centre and $C_2 O$ as a radius the other of these relative valve circles, the intersections U_2 and U_0 will fall upon the aforesaid crank directions. Moreover, if we connect the centres C_1 and C_2 with the centre of the eccentricity O D of the main valve, and draw through O the lines $O E_1$ and $O E_2$ parallel to the connecting lines, then the points E_1 and E_2 will be the limits between which the end D_0 of the eccentricity of the expansion eccentric must be shifted along the line $M_0 M_0$ by the assumed mechanism, in order that *all possible* grades of expansion may be attained.

This examination also completes the presentation of the diagram as applied to *Guinotte's* valve-gears; but perhaps one point more should be mentioned, namely, that the second intersection of the circle having s as a radius, with the relative valve circle described

about C_2, must of course fall *behind* crank position 1, otherwise the expansion-valve would reopen the steam-way before the main valve had closed the cylinder-port.

How the centre of the relative valve circle is to be found on the relative line of centres N N, and how the position $O D_0$ of the expansion eccentric is to be determined for a given ratio of expansion, is so evident from what has been said, that further explanation is unnecessary.

On the other hand, we will now explain how *Guinotte's* mechanism varies the eccentricity of the expansion eccentric so that the end D_0 can travel on the straight line $M_0 M_0$ between the limits E_1 and E_2.

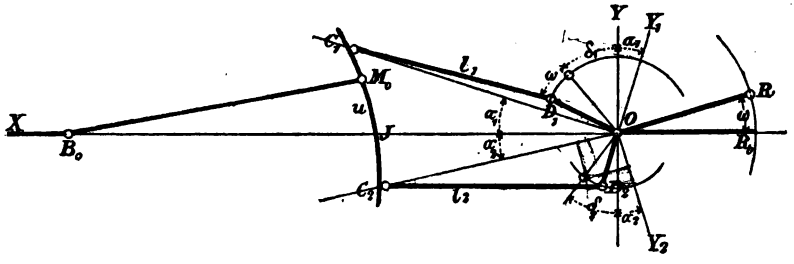
One Form of Guinotte's Cut-off Gear.—If we carefully examine the arrangement of *Polonceau's* gear, Fig. 25, Pl. V., and the corresponding diagram Fig. 27, the thought is suggested that the straight line $M_0 M_0$ (Fig. 53), which coincides with the straight line $D D_0$ (Fig. 27, Pl. V.) in *Polonceau's* gear, can be inclined to the axis O X by an *unsymmetrical* construction of *Gooch's* link-motion (used by *Polonceau* to give motion to the expansion-valve), that is to say, by giving to the eccentricities, angles of advance, eccentric-rods and even to the parts of the link on opposite sides of its dead point, *different* values for the forward and back eccentric. It appears also from *Guinotte's* writings that this was the line of thought which led him to his new valve-gear arrangements, for his investigations were preceded by the considerations now to be given, with this difference, that we treat *Gooch's* gear, thus modified, *analytically*. In doing this we might proceed as in the analytical investigation of *Stephenson's* and *Gooch's* valve-gears, but can attain our object more easily by an approximate procedure recently outlined by *Pichault* * ; at least we start with the same fundamental thought in the following analytical investigation.

Let us suppose the expansion-valve to be driven by the radius-rod $M_0 B_0$ (Fig. 54) and the link $C_1 C_2$, the latter being curved to

* *Pichault*, "Étude sur les appareils de distribution de vapeur, exposés à Vienne en 1873," 'Annales industrielles,' 6e Année, 1874. See literary notices at end of book.

the radius $B_0 M_0$. Let the point J be taken as the dead point, and by suitably suspending the link, let this point be guided in the line of stroke O X; let the block M_0 be at the distance $J M_0 = u$ from the dead point of the link. Let the expansion be varied by raising and lowering the radius-rod, i. e. by varying u, let us suppose the ends

FIG. 54.



C_1 and C_2 of the unequal eccentric-rods $D_1 C_1 = l$, and $D_2 C_2 = l_2$ to be at the distances $J C_1 = c_1$ and $J C_2 = c_2$ from the dead point of the link; let the two eccentrics have the eccentricities $O D_1 = r_1$ and $O D_2 = r_2$, the latter making with the perpendicular O Y to the direction of the stroke, the angles of advance δ_1 and δ_2 shown in the figure.

The problem now is to determine the equation for the movement of the valve, or, what amounts to the same thing, the movement of the point B_0.

Let us draw the line $O C_1$ and suppose the link to be detached from the rod at C_1, and let us regard $O C_1$ as the direction of the stroke, by supposing that at first only the eccentricity $O D_1$ turns through the angle ω; the point C_1 will then oscillate about a certain centre of motion like the valve of a simple valve-gear; if we let fall on the assumed stroke $O C_1$, the perpendicular $O Y_1$, which makes with O Y an angle a_1 equal to the inclination of $O C_1$ to O X, the angle $\delta_1 + a_1$ will be equivalent to an angle of advance, and the movement of the point C_1 from its middle position for an angular motion ω, will be

$$r_1 \sin (\delta_1 + a_1 + \omega).$$

Now if we suppose the link to be again joined to the rod at C_1, but the other point C_2 to be stationary, so that the link will swing about the fixed point C_2, the movement of the point C_1 will be accompanied by a movement of the link-block M_0, and consequently by a movement of the end B_0, the latter movement being smaller in the ratio

$$\frac{C_2 M_0}{C_2 C_1} = \frac{c_2 + u}{c_2 + c_1};$$

this movement of B_0, which we will designate by ξ', is therefore

$$\xi' = \frac{c_2 + u}{c_2 + c_1} r_1 \sin(\delta_1 + a_1 + \omega). \tag{112}$$

In like manner we may now suppose the eccentricity $O D_1$ and the point C_1 to be temporarily fixed and the eccentricity $O D_2 = r_2$ to turn through the angle ω. Now if the line $O C_2$ be taken as the direction of stroke, we shall have for this simple gear $(\delta_2 + a_2)$ as an angle of advance, and for the movement of point C_2 from its middle position,

$$r_2 \sin(\delta_2 + a_2 - \omega).$$

As the link now swings about the fixed point C_1, the movement ξ'' of the link-block will be smaller in the ratio

$$\frac{C_1 M_0}{C_1 C_2} = \frac{c_1 - u}{c_2 + c_1},$$

and therefore

$$\xi'' = \frac{c_1 - u}{c_2 + c_1} r_2 \sin(\delta_2 + a_2 - \omega). \tag{113}$$

In reality, however, the two motions, which we have here considered separately, take place simultaneously, hence the total movement of the block M_0 and therefore of the expansion-valve from the middle of its stroke will be

$$\xi_0 = \xi' + \xi'';$$

substituting the values given by equations (112) and (113), and as before designating the whole length of link $C_1 C_2 = c_2 + c_1$ by $2c$, we have

$$\xi_0 = \frac{c_2 + u}{2c} r_1 \sin(\delta_1 + a_1 + \omega) + \frac{c_1 - u}{2c} r_2 \sin(\delta_2 + a_2 - \omega). \tag{114}$$

In this equation for the travel of the expansion-valve there must still be determined the angles a_1 and a_2, all other values being given with the valve-gear. These angles may be eliminated by the following approximate method. If we assume long eccentric-rods and a comparatively short link, the two angles a_1 and a_2 (Fig. 54) will be very small and their cosines can each be taken equal to unity, and we may approximately make $O C_1 = D_1 C_1 = l_1$ and $O C_2 = D_2 C_2 = l_2$, and therefore assume

$$\sin a_1 = \frac{c_1}{l_1}$$

and

$$\sin a_2 = \frac{c_2}{l_2};$$

developing the sines in equation (114) and substituting the preceding expressions, the equation for the movement of the valve reduces to the form

$$\xi_0 = A \cos \omega + B \sin \omega, \tag{115}$$

the values of A and B being given by the following equations:

$$A = \frac{(c_2 + u)}{2c} r_1 \left(\sin \delta_1 + \frac{c_1}{l_1} \cos \delta_1\right) + \frac{c_1 - u}{2c} r_2 \left(\sin \delta_2 + \frac{c_2}{l_2} \cos \delta_2\right). \tag{116}$$

$$B = \frac{(c_2 + u)}{2c} r_1 \left(\cos \delta_1 - \frac{c_1}{l_1} \sin \delta_1\right) - \frac{(c_1 - u)}{2c} r_2 \left(\cos \delta_2 - \frac{c_2}{l_2} \sin \delta_2\right). \tag{117}$$

These equations at once show that the investigated mechanism is really useful in the sense explained above; the values of A and B, as shown in the first section of the book, are the co-ordinates of the end of the diameter of the valve circle; if, in a given gear and for a given position u of the block in the link, we were to calculate the values of A and B and lay off (Fig. 53) $O A = A$ and $A D_0 = B$, we should get in the distance $O D_0$, the corresponding eccentricity of the expansion-valve. According to the equations (116) and (117), a variation of u is accompanied by a variation in the values of A and B and in the position of D_0. But as both equations contain u to the first power, it at once follows that the point D_0 traverses a straight line $M_0 M_0$ when u varies, i.e. when the radius-rod is raised and lowered. If we transform this gear (Fig. 54) into that of *Gooch* by

making $r_1 = r_2 = r$, $\delta_1 = \delta_2 = \delta$, $l_1 = l_2 = l$, and $c_1 = c_2 = c$, the two equations (116) and (117) become

$$A = r\left(\sin\delta + \frac{c}{l}\cos\delta\right)$$

$$B = \frac{u\,r}{c}\left(\cos\delta - \frac{c}{l}\sin\delta\right),$$

which values are like those found on p. 112; in this case A is independent of u, that is, it is constant; the line $M_0 M_0$ (Fig. 53) is therefore perpendicular to O X, as in *Polonceau's* gear. Consequently *Gooch's* arrangement *must* be constructed *unsymmetrically*, as stated by *Guinotte*, if the straight line $M_0 M_0$ is to cut both axes of co-ordinates (as in Fig. 53); it is easy to determine this line for a given valve-gear, for it is only necessary to determine the position of D_0 for two different values of u and to connect these positions by a straight line. But the inverse problem, to determine the dimensions from the course of the given straight line, is a difficult one. To solve this problem, it is easiest to employ an approximate method; if the eccentric-rods are very long, and this was assumed in developing the above formulas, the influence of the terms containing the factors $c_1 : l_1$ and $c_2 : l_2$ is so small that they can be neglected, and we then have the simpler equations

$$A = \frac{(c_2 + u)}{2\,c}\,r_1 \sin\delta_1 + \frac{(c_1 - u)}{2\,c}\,r_2 \sin\delta_2, \qquad (118)$$

$$B = \frac{(c_2 + u)}{2\,c}\,r_1 \cos\delta_1 - \frac{(c_1 - u)}{2\,c}\,r_2 \cos\delta_2. \qquad (119)$$

Now if we suppose the radius-rod to be raised to C_1 (Fig. 54), i. e. make $u = c_1$, the preceding formulas will give for A and B the values

$$A_1 = r_1 \sin\delta_1$$

and

$$B_1 = r_1 \cos\delta_1,$$

but these are the co-ordinates of the end D_1 of the first eccentric $O D_1$ when the crank is at the dead point. On the other hand, if the radius-rod is placed at C_2, that is, $u = -C_2$, and if we designate

the corresponding values of A and B by A_2 and B_2, we shall have from equations (118) and (119)

$$A_2 = r_2 \sin \delta_2$$

and

$$B_2 = -r_2 \cos \delta_2 .$$

But these are again the co-ordinates of the end D_2 of the eccentricity of the second eccentric $O D_2$ when the crank passes through the dead point. This gives us the following simple proposition:—

Draw the two eccentricities $O D_1$ and $O D_2$ (Fig. 55) with the given angles of advance and connect their ends by a straight line $D_1 D_2$; this line is then the line $M_0 M_0$, which we have discussed in connection with the diagram of *Guinotte's* gear. *Guinotte* and *Pichault* have developed this proposition by a different method and regard it as exact; it serves, however, only as a first approximation. If, in the argument leading to this proposition, we had made use of equations (116) and (117) we should have obtained more exact values for these co-ordinates, namely,

$$A_1 = r_1 \left(\sin \delta_1 + \frac{c_1}{l_1} \cos \delta_1\right)$$

$$B_1 = r_1 \left(\cos \delta_1 - \frac{c_1}{l_1} \sin \delta_1\right),$$

and

$$A_2 = r_2 \left(\sin \delta_2 + \frac{c_2}{l_2} \cos \delta_2\right)$$

$$B_2 = -r_2 \left(\cos \delta_2 - \frac{c_2}{l_2} \sin \delta_2\right),$$

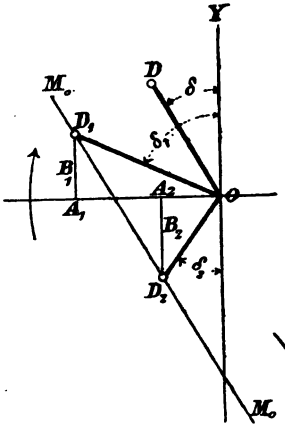
or if in developing the proposition we had gone back to equation (114), we should get

$$A_1 = r_1 \sin (\delta_1 + a_1) \qquad B_1 = r_1 \cos (\delta_1 + a_1)$$
$$A_2 = r_2 \sin (\delta_2 + a_2) \qquad B_2 = -r_2 \cos (\delta_2 + a_2),$$

where a_1 and a_2 have the meaning given by Fig. 54. The *exact* position of the line $M_0 M_0$ (Fig. 55) could easily be found from the formulas just given, but we need not follow the matter further. The preceding approximate calculation shows, moreover, that in *Guinotte's* gear the two eccentric-rods may be made equal and the link suspended *at the centre*, as is done by *Gooch*, only the two eccentricities

r_1 and r_2 and the angles of advance δ_1 and δ_2 being taken different. In designing a new valve-gear we first draw the diagram for the main valve; then, with the help of the data, locate the relative curve of centres N N (Fig. 53), and finally draw the guiding line M_0 M_0. As a first approximation we now choose the two eccentricities so that their ends D_1 and D_2 (Fig. 55) fall on the line M_0 M_0. The form of *Guinotte's* gear above described is only applicable to engines which always run in one direction; this form cannot therefore be employed as a reversing gear, and in this respect it is inferior to *Polonceau's* gear.

FIG. 55.



Second Form of Guinotte's Gear.—The valve-gear now to be examined has been constructed by *Guinotte* in various ways, and solves in a very ingenious and elegant manner the problem stated when discussing the diagram (Fig. 53). An important point in connection with this simple mechanism is that it answers for both the forward and the reverse motion of the engine, and is therefore applicable to locomotive, marine, and hoisting engines.

Let us first suppose that Fig. 29, Pl. VI., represents the gear applied to a non-reversible, stationary engine. The main eccentric is keyed to the shaft O with the angle of advance Y O D and drives the main valve S by means of the radius-rod D Q.

A second eccentric O D_0 is keyed to the shaft with an angle of advance of 90° and, by means of the eccentric-rod D_0 J, drives the link C C_1 which turns about the point J. This pin J is also attached to the end of the lever J K L which turns about the *fixed* axis K; a second lever F Q E has its fulcrum at the end Q of the stem D Q of the main valve. By means of the coupling rods L F and E C the motion of the end L of the first lever is transmitted to the second one, and from this lever (whose fulcrum Q has the same motion as the main valve) to the end C of the link, the motions imparted to

the points J and C giving to the link a peculiar reciprocating and swinging motion.

The block M can be shifted in the curved slot of the link, and the radius-rod B M, connected with the link, is attached at B to the stem of the expansion-valve S_0. The expansion-valve is like *Polonceau's*, which is shown in Fig. 23, Pl. V.

We will now develop analytically the theory of this valve-gear by determining the equation for the movement of the expansion-valve; that is, we will develop the theory differently from *Guinotte*, whose discussions were of a more graphical character.

The movement of the link $C C_1$ (Fig. 29, Pl. VI.) is a very complicated one, but can be easily represented analytically, if we follow its different motions separately. I shall first assume that the lever J K L is detached from the middle J of the link and that this point J is driven by an eccentric $O D_0$, whose eccentricity is r_0 and angle of advance δ_0, and shall also assume that the main eccentric O D is likewise held fast temporarily; now if the eccentric $O D_0$ turns through the angle ω, the point J of the link will shift a distance ξ_1 to the left,* and the link will turn about its *fixed* end C, for, under these suppositions, neither the lever J K L nor the lever F Q E will move. If we designate the distance C J by c and if the block M is at the distance $J M = u$ from the middle of the link, the block, the end B of the radius-rod and the expansion-valve S_0 will all shift to the left a distance ξ', whose ratio to the movement ξ_1 of the point J is as C M : C J, or since $C M = c - u$,

$$\xi' = \frac{c - u}{c} \xi_1 . \qquad (120)$$

Now let both eccentrics O D and $O D_0$ be held fast and let the lever J K L be turned till its end J again coincides with the point J which has been shifted a distance ξ_1; let the points be again connected. If we designate the lever arms J K and K L by a and b, the assumed turning of the lever causes its end L and also the end F of the second lever F Q E to shift to the right a distance $\frac{b}{a} \xi_1$, and this causes the

* [In saying this motion is to the left the general case is assumed that $\delta_0 < 90°$, in reality for the gear shown in Fig. 29, Pl. VI., $\delta_0 = 90°$.—Ed.]

end E of the second lever and the end C of the link (now turning about its fixed centre J) to shift to the left a distance

$$\frac{b_1}{a_1}\frac{b}{a}\xi_1,$$

where a_1 and b_1 represent the lever arms F Q and Q E. But this second turning of the link causes the block M and consequently the expansion-valve S_0 to shift to the left a distance ξ'', which may be found by multiplying the preceding value by the ratio M J : C J or $u : c$, which gives

$$\xi'' = \frac{u}{c}\frac{b}{a}\frac{b_1}{a_1}\xi_1. \tag{121}$$

Finally, we must consider the third and last motion: the point J of the link and the lever J K L are fixed, and we must imagine the eccentricity O D $= r$ of the main valve to be turned through the angle ω. The consequence is that the point Q is shifted with the main valve a certain distance to the left, and as the lever F Q E now turns about the fixed point F, its end E and the point C of the link must move to the left a distance

$$\frac{a_1 + b_1}{a_1}\xi;$$

the block M and with it the expansion-valve S_0 must therefore traverse a third distance ξ''' expressed by

$$\xi''' = \frac{u}{c}\frac{a_1 + b_1}{a_1}\xi. \tag{122}$$

The various parts of the mechanism are now evidently in the same position as if the three separately executed motions had taken place simultaneously. The total movement ξ_0 of the expansion-valve is therefore

$$\xi_0 = \xi' + \xi'' + \xi''',$$

or substituting the values given by equations (120) to (122) we have

$$\xi_0 = \left(\frac{c-u}{c} + \frac{u}{c}\frac{b}{a}\frac{b_1}{a_1}\right)\xi_1 + \frac{u}{c}\frac{a_1 + b_1}{a_1}\xi.$$

But according to the assumed notation, the distances estimated from the middle position are

$$\xi = r \sin(\delta + \omega),$$

and

$$\xi_1 = r_0 \sin(\delta_0 + \omega),$$

or if we assume with *Guinotte*, the angle of advance δ_0 of the eccentric $O D_0$ to be 90°, we get

$$\xi_1 = r_0 \cos \omega.$$

If these expressions are substituted in the preceding value of ξ_0, we shall have after a few simple transformations, an expression for the movement ξ_0 of the expansion-valve, of the well-known form,

$$\xi_0 = A \cos \omega + B \sin \omega, \tag{123}$$

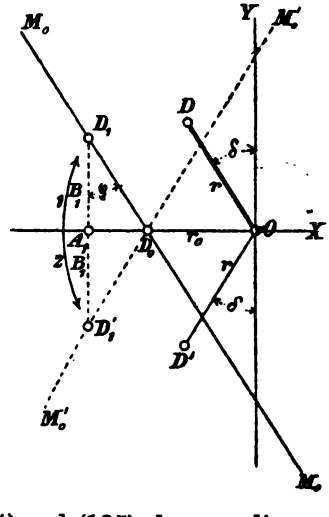
where

$$A = r_0 + \frac{u}{c}\left[\frac{a_1 + b_1}{a_1} r \sin \delta - \left(1 - \frac{b}{a}\frac{b_1}{a_1}\right) r_0\right]. \tag{124}$$

$$B = \frac{u}{c}\frac{a_1 + b_1}{a_1} r \cos \delta. \tag{125}$$

As A and B represent the co-ordinates of the end D_0 of the eccentricity $O D_0$ of the expansion eccentric, and as the co-ordinates vary with u, these equations will enable us to determine the location of the line $M_0 M_0$, two points being sufficient for this purpose.

Fig. 56.



To determine one point let $u = 0$ and designate the corresponding values of A and B by A_0 and B_0, then, according to equations (124) and (125), we have

$$A_0 = r_0 \quad \text{and} \quad B_0 = 0,$$

consequently the line passes through the end D_0 (Fig. 56) of the eccentricity r_0. On the other hand, if we make $u = c$, that is, raise the radius-rod to the point C (Fig. 29, Pl. VI.) of the link, and designate the corresponding values of A and B by A_1 and B_1 we shall get from equations (124) and (125) the co-ordinates of a second point D_1 of the line $M_0 M_0$, namely,

$$A_1 = \frac{a_1 + b_1}{a_1} r \sin \delta + \frac{b}{a}\frac{b_1}{a_1} r_0 \tag{126}$$

$$B_1 = \frac{a_1 + b_1}{a_1} r \cos \delta. \tag{127}$$

As soon as this line is determined for a given valve-gear, there will follow all that was said of Fig. 53 on p. 216 concerning the distribution of steam. We may now, by working backwards, readily determine certain dimensions of the mechanism when the guiding line $M_0 M_0$ is given. We will call attention to the particular case in which the lever arms of the two levers J K L and F Q E bear to each other the relation

$$a : b = b_1 : a_1,$$

a relation that is easily realized practically; this proportion is equivalent to

$$\frac{b\, b_1}{a\, a_1} = 1,$$

which reduces equation (126) to

$$A_1 - r_0 = \frac{a_1 + b_1}{a_1}\, r \sin \delta.$$

But this distance is represented in Fig. 56 by $A_1 D_0$. Now according to the figure the inclination ϕ of the line $M_0 M_0$ to the vertical is given by

$$\tan \phi = \frac{A_1 D_0}{A_1 D_1}, \quad = \frac{a_1 + b_1}{a_1} \cdot r \sin \delta =$$

which in our notation is equivalent to

$$\frac{a_1 + b_1}{a_1} \cdot r \cos \delta$$

$$\tan \phi = \frac{A_1 - r_0}{B_1}. \quad = \tan \delta$$

Substituting in this equation the above value for $A_1 - r_0$ and the value of B_1 given by equation (127), we get

$$\tan \phi = \tan \delta \quad \text{or} \quad \phi = \delta.$$

This gives us the interesting proposition that, with the assumed leverages, the line $M_0 M_0$ is *parallel* to the eccentricity O D of the main eccentric; in this particular case, therefore, we may determine the position of $M_0 M_0$ without any calculation whatever.

Before closing we must call attention to a matter of controlling importance. If the main valve is driven by the eccentric O D, the engine will run in one, say, the *forward* direction, represented by arrow No. 1 (Fig. 56). Now if the engine is to be reversed, either

the eccentric O D must be shifted to the position O D′ or the main valve must be driven by another eccentric O D′. But this is equivalent to changing the angle of advance δ of the eccentric O D to the value $(180° - \delta)$. Substituting this value in equations (126) and (127) we find that A_1 is unchanged in direction and magnitude, but that while the numerical value of B_1 remains the same, *its sign changes;* consequently reversing will bring the point D_1 to D_1', and the line $M_0 M_0$ will shift to $M_0' M_0'$, the latter line occupying the same position relatively to the back eccentric that $M_0 M_0$ occupied relatively to the forward eccentric.

From this follows the remarkable proposition, that with this gear the reversing of the engine can be effected without the locomotive runner altering anything in the mechanism that controls the motion of the expansion-valve.

Guinotte rightly emphasises this great advantage of his valve-gear, and indeed we must regard his invention as a very important advance in the construction of cut-off gears.

As regards the reversing gear, it is here perfectly immaterial whether the main valve is driven by *Stephenson's, Gooch's, Allan's,* or *Heusinger v. Waldegg's* link-motion, for *Guinotte's* gear can be combined with any one of these; according to *Guinotte* the eccentric O D_0 (Fig. 29, Pl. VI.) can be dispensed with, if, as in *H. v. Waldegg's* gear, the crosshead sets in motion a two-armed lever whose second end guides the centre J of the link. Indeed, it is not difficult to transform *Guinotte's* gear in various ways. Several such transformations are given in the article by *Guinotte* already mentioned.

CHAPTER IV.

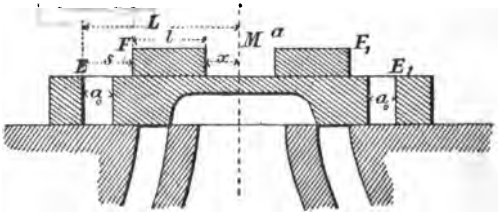
MEYER'S CUT-OFF GEAR.

Meyer's gear first came into use in 1842; only its valve arrangement is shown in Fig. 28, Pl. V. The main valve, as in *Polonceau's* gear, is provided with two steam-ways $a_0\, a_0$, and carries the expansion-valve which here has the form of *two* plates $S_0\, S_0$. These plates also

act as nuts to the right- and left-hand screws *h h* cut on different portions of the valve-stem B B. Consequently, if the valve-stem is turned about its longitudinal axis, the plates will recede from or approach each other according to the direction in which the stem is turned. In a certain sense, therefore, the expansion-valve is a plate that can be lengthened or shortened at will, thus effecting, as future investigations will show, an earlier or later cut-off. The variation of the expansion by turning the valve-stem B B, can also be effected during the running of the engine and in various ways.

The arrangement shown in Fig. 28 is easily understood. The bracket T carries the sleeve V in which one end of the valve-stem slides. When the stem is turned by the hand wheel W, a sleeve-shaped screw surrounding the end of the stem also turns, but takes no part in the sliding motion of the stem; the screw on the outside of this sleeve works in a nut, on the outer surface of which there is a visible index line *a*, whose position along the graduated scale, marked on the fixed sleeve V, shows at any instant the position of the valve-plates S_0 S_0 in the interior of the steam-chest, and thus indicates the degree of expansion with which the gear is working. The setting of both eccentrics is shown in Fig. 22*a*, and is like that in *Bréval's* and in *Polonceau's* gear. $O D = r$ is the eccentricity of the main eccentric, and $O D_0 = r_0$ that of the expansion eccentric, the latter being set with an angle of advance $Y O D_0 = \delta_0$. In Fig. 57 both

Fig. 57.



valves are again shown and in their middle positions; the position here occupied by the valves relatively to each other and to the cylinder is of course only possible when the valve-rods are detached from their eccentrics.

Let us first assume that we are dealing with a *Meyer* gear for a non-reversible, stationary engine, then the diagram of Fig. 24, Pl. V., discussed on p. 204 and p. 209 in connection with *Polonceau's* gear, is also applicable here. If, therefore, in a *Meyer* gear, the eccentricity of the main eccentric is $r = 0 \cdot 040^m$ $(1 \cdot 57'')$ and that of the expansion eccentric is $r_0 = 0 \cdot 045^m$ $(1 \cdot 77'')$ while the angle of advance of the former is $\delta = 15°$ and that of the latter is $\delta_0 = 85°$, we lay off these values in the now well-known manner (Fig. 24, Pl. V.), construct the parallelogram $O D_0 D D_x$ and describe on its side $O D_x$, representing the relative eccentricity, the valve circle that expresses the law of motion of the upper valve relatively to the lower one.

We now describe, with $E F = s$ as a radius, a circle $U_0 U U_1$ about O and find its intersections U_1 and U_2 with the relative valve circle; in the figure the value $s = 0 \cdot 030^m$ $(1 \cdot 18'')$ is assumed. The crank directions 5 and 6 passing through the points U_1 and U_2, correspond respectively to the beginning of expansion and the reopening of the steam-way. Thus far, therefore, there is no difference between *Polonceau's* and *Meyer's* gears; no difference appears till the method of varying expansion comes up. In *Polonceau's* gear the quantity s is constant and the relative eccentricity is variable in magnitude and direction, while in *Meyer's* gear just the opposite is true. The influence of the variation of the value $E F = s$ (Fig. 57) is easily seen from the diagram; the smaller this value, i. e. the farther apart the plates, the more crank position 5 is shifted to the left and the earlier does the cut-off take place. The plates may even be separated so far that the edge F will fall to the left of E (Fig. 57); the intersections U_1 and U_2 then interchange their significations, as was explained on p. 186. *Meyer's* gear can easily be designed so as to effect all possible degrees of expansion from zero on; of the valve-gears hitherto examined only *Georges'* and *Guinotte's* possess this property; to obtain this result with *Meyer's* gear it is necessary to make the eccentricities r and r_0 unequal as will be shown later on. It is usual in practice to make the eccentricities equal, and we will give an example of this case as it will afford a favourable opportunity of discussing the most important points.

In the *Meyer* gear let the eccentricities of the main and expan-

sion eccentric be $r = r_0 = 0 \cdot 040^m$ $(1 \cdot 57'')$, let the angle of advance of the former be $15°$ and that of the latter $\delta = 60°$. The expansion-valve is now to be arranged so that any degree of expansion, $\cdot 0$, $0 \cdot 1$, $0 \cdot 2$ to $0 \cdot 8$, can be obtained at will; the problem is to determine the corresponding positions of the plates and the requisite dimensions of the various parts.

Lay off (Fig. 30, Pl. VI.) the given eccentricities in the usual manner and draw the relative valve circle on the diameter $O D_\infty$, then draw $L_0 L$ parallel to $O X$, consider this line as the piston stroke and determine the piston positions for each of the given ratios of expansion; we have assumed the distance $L_0 L = 0 \cdot 10^m$ $(3 \cdot 94'')$; the crank radius $O R_0$ of the crank-pin circle $R_0 R_1 R_2$ is therefore equal to $0 \cdot 050^m$ $(1 \cdot 97'')$. From the piston positions we next determine the corresponding crank positions $O R_1$, $O R_2$, &c., and find their intersections U_1, U_2, U_3, &c., with the relative valve circle. The distances $O U_1$, $O U_2$, &c., at once give the required values of $E F = s$ (Fig. 57) for the various ratios of expansion, and these values of s are here given full size because the eccentricities were drawn full size. For example, the ratio of expansion $0 \cdot 4$ has the crank position $O R_4$, and measurement shows the corresponding value of $O U_4 = s$ is $13 \cdot 1^{mm}$ $(0 \cdot 52'')$. For the ratio of expansion $0 \cdot 1$ we have the crank position $O R_1$, and the distance $O U_1 = 9 \cdot 5^{mm}$ $(0 \cdot 36'')$ is negative, i. e. for this ratio of expansion the plates are so far separated that the edge F (Fig. 57) lies $9 \cdot 5^{mm}$ to the *left* of E (when both valves are in their middle positions). When the plates are separated so far that F is at the distance $O U_0 = 24 \cdot 5^{mm}$ $(0 \cdot 96'')$ to the left of E, the degree of expansion is zero, i. e. the valve cuts off at the beginning of the stroke. In the lower part of Fig. 30, Pl. VI., the various values are laid off on a straight line, the positive values lying to the left of E and the negative ones to the right; the positions of the edge F relatively to E are then at once seen for each ratio of expansion. But with the dimensions assumed for Fig. 30, Pl. VI., there is a certain limit to the ratio of expansion in one direction. If we separate the plates so far that $s = O U_0$, we shall get the ratio of expansion $0 \cdot 8$; if we continue the circle described with $O U_0$ till we obtain the second intersection

U_0', then $O R_0'$ is the crank position at which the steam-way reopens, and, as this position is *behind* position 1 at which the main valve closes (the outside lap of the latter is $O V = e = 0 \cdot 008^m$ ($0 \cdot 31''$), this position of the plates is still a permissible one; but we see from this that the greatest permissible value of s is given by $O U' = O U$, U representing the intersection of crank position 1 with the relative valve circle. At this limiting crank position $O R'$ the ratio of expansion is $0 \cdot 830$, and the expansion-valve reopens the steam-way at the same instant that the main valve closes the cylinder-port. If the plates were separated till the distance s fell between the values $O U$ and $O D_x$, steam would enter twice during the same stroke, for the upper valve would reopen the steam-way in the main valve before the latter had closed the cylinder-port. If the distance s were taken *greater* than $O D_x$ the expansion-valve would become useless, for it would no longer close the steam-way, but simply contract and widen its port alternately. Moreover, we see that it is advantageous to allow the relative eccentricity $O D_x$ to coincide in direction with crank position 1; but this generally requires that the two eccentricities $O D$ and $O D_0$ be different, i. e. that two different eccentrics be employed.

In the present example it will be best to take the greatest value of s equal to $O U = O U' = A E = 29 \cdot 5^{mm}$ ($1 \cdot 16''$), and for this value let the two plates (Fig. 57) touch each other at the middle; the value represented by x is then zero, and if we designate this greatest value $O U = A E = s$ by s_1 and use the notation employed with Fig. 57, we obtain

$$L = l + s_1. \tag{128}$$

Then if we wish to vary the expansion down to zero, the other limit of s which we will designate by s_2, will be given by $O U_0 = E B$, that is the plates will be separated till the edge F is at the left of E when the valves are at their middle positions. Now when the upper valve moves on the lower one (its greatest relative movement to the left is $O D_x = r_x$) its right edge must not pass over the steam-way; we see therefore that the length l of the plate cannot be less than that given by the equation

$$l = r_x + s_2 + a_0. \tag{129}$$

If, for the present case, we assume the width of the steam-way to be $a_0 = 30^{mm}$ (1·18″) and take from Fig. 30, Pl. VI., the values $O D_x = r_x = 31^{mm}$ (1·22″) and $O U_0 = s_2 = 25^{mm}$ (0·99″), the minimum length of the expansion-valve is $l = 86^{mm}$ (3·49″). As the greatest positive value of s was found to be $O U = s_1 = 29 \cdot 5^{mm}$ (1·16″), we find from equation (128) the length L which is a dimension of the main valve (Fig. 57), to be

$$L = 115 \cdot 5^{mm} \ (4 \cdot 55'').$$

Both values L and l are minimum values which in practice will be increased by a few millimetres. It follows from what has preceded that the lower part of Fig. 30, Pl. VI., gives, full size, the distance $A F = x$ (Fig. 57), which represents the amount that each plate must be shifted from the centre to attain the assumed ratio of expansion.

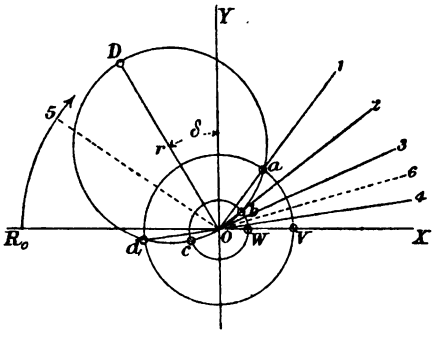
The preceding investigations apply only to non-reversible engines, but they can be readily transferred to reversible engines. For investigating the reverse motion we must, in constructing Fig. 30, Pl. VI., use in place of the eccentricity O D of the forward eccentric, that of the back eccentric. If we make the angle of advance of the expansion eccentric $\delta_0 = 90°$, the diagram gives equal results for the forward and reverse motion. But in locomotive work, for obvious reasons, attention should principally be directed to the forward motion and the angle of advance, for the expansion eccentric should be taken less than 90°; the effect of this is to bring the relative eccentricity $O D_x$ (Fig. 30, Pl. VI.) nearer to crank position 1, thus increasing, as shown above, the limits of expansion. The arrangement is then less perfect for the reverse motion, but this is not of much consequence in locomotives that seldom run backward.

CHAPTER V.

Napier and Rankine's Cut-off Gear.

In the introductory remarks to the investigations on double valve-gears, attention was called to the fact, that link-motions, with but *one* valve, exhibit the peculiarity of earlier release and compression when the expansion is increased. If we suppose the block to be so placed in the link that the resulting valve-motion is as if an eccentric with eccentricity O D and angle of advance δ (Fig. 58) were in action, the four principal crank positions 1 to 4 will be given by the intersections a, b, c, d with the lap circles. Now, link-motions effect an increase of expansion by simultaneously varying r and the angle of advance δ. The figure at once shows that increasing δ not only shifts position 1 nearer to D, but also shifts the remaining positions 2, 3, and 4 in the same direction; but the shifting of the last three is not desirable, and this is particularly true of positions 2 and 3, which represent the beginning of the compression and of the release; either these positions should remain invariable, as in all the double valve-gears considered above, or they should shift but slightly when positions 1 and 4 change. The latter result is attained by *Napier* and *Rankine's* valve arrangement shown in Fig. 31, Pl. VI.; strictly speaking, this gear possesses no expansion-valve which slides on the main valve, for the latter S slides on a movable valve seat. Under the main valve lies a frame-like valve S_0 S_0, whose stem B_0 is driven by a special eccentric O $D_0 = r_0$, set with an angle of advance of 90° (Fig. 31*a*). This eccentricity r_0 is small and gives to the induction

Fig. 58.



edges of the cylinder-ports a slight reciprocating motion. In Fig. 31 both valves are shown in their middle positions; the distance between the edges F and E represents the outside lap e; the inside lap i is not specially emphasized in the figure, but has here exactly the same meaning as before (see Fig. 7, p. 25). The action of this gear, as *Napier* and *Rankine* have shown, can be easily followed with the help of one diagram. Fig. 32, Pl. VI., represents such a diagram.

Let $C C_1$ represent the curve on which the centres of the valve circles travel when the block is shifted in the link, and for the present, let us suppose the block so placed that the valve moves as if it were driven by the eccentric O D. Now describe the valve circle on O D as a diameter, draw the inside lap-circle, and find the intersections b and c. If we connect these points with O, crank positions 2 and 3 will give us the beginning of the compression and of the release. Thus far the procedure is the same as the ordinary one, but the determination of positions 1 and 4 is different, for the induction edge E is movable; the problem here is to determine the motion of the edge F relatively to the edge E; but the latter is driven by the eccentric $O D_0$ and the former by O D, consequently, if we construct on O D as a diagonal the parallelogram $O D_0 D D_x$, we shall have $O D_x$ for the relative eccentricity, and the circle described on the latter as a diameter for the relative valve circle. Describing the outside lap-circle, the intersections a and d will give the required crank positions 1 and 4, the former corresponding to the beginning of expansion and the latter to the beginning of admission at the other end of the cylinder. If the valve S_0 were stationary and the outside lap e were the same as before, crank positions 1 and 4 would lie at 1′ and 4′, and this shows that *Napier* and *Rankine* attained the object mentioned: expansion is varied while 2 and 3 remain the same. Moreover, the diagram shows us that the curve of centres $C' C_1'$ of the relative valve circle is parallel to the original curve $C C_1$ but at a distance $\frac{1}{2} r_0$ to the left of it.

Napier and *Rankine's* arrangement is without doubt an ingenious one, but it will probably not be extensively employed on locomotives; after all, the principal object of the gear is not attained, and when the expansion is very great, i. e. when the block is placed nearer the

dead point of the link, it but imperfectly fulfils its purpose, for the two crank positions 2 and 3 approach nearer to the eccentricity O D; an earlier compression and release, therefore, still accompanies increased expansion, though to a less degree.

CHAPTER VI.

FARCOT'S CUT-OFF GEAR.

FARCOT'S gear, first used in 1838, is most often found on French engines. The form of the main valve is similar to that in *Polonceau's* and *Meyer's* gears, i. e. it is provided with steam-ways opened and closed by an expansion-valve which slides on the back of the main valve. But the gear differs essentially from those hitherto treated in that the expansion-valve is loose on the main valve, receiving no motion whatever from the outside. In consequence of friction between it and the main valve, it is carried (dragged) back and forth by the latter, but is prevented from moving with the main valve at certain proper points of the latter's travel. This gear does not need any special analysis, our diagram easily and briefly explaining the action of the expansion-valve; here the problem is simply to determine the instants at which cut-off takes place and the steam way is reopened, and to determine the limits between which expansion varies. Farcot's gears may be divided into two classes: one in which the expansion-valve consists of only one piece and another in which the expansion-valve consists of two pieces; we will treat these two classes separately

a. Gear with a One-part Expansion-valve.

Fig. 33, Pl. VI., shows the valve arrangement of such a gear. On the main valve S there is a gridiron plate S_0 S_0 which has two lugs *n n* and acts as an expansion-valve. Between these lugs is a stop K at the lower end of an arbor A that passes through a stuffing box in the cover of the steam-chest. The form of the stop

is shown at K_1 and can be brought into different positions by the arbor A, and in this way its diameter in the direction of the valve-motion may be varied.

Now, if we suppose the lower valve to be shifted to the right (we have supposed both valves to be at first in their middle positions) the upper valve will move with the lower one till the left lug strikes the stop; the upper valve then stands still while the main valve continues its motion; consequently the plates of the expansion-valve pass over the steam-ports in the main valve, contracting the latter and then closing them at a certain moment.

If the main valve moves to the left it takes the expansion-valve with it till the right-hand lug strikes the stop; then the upper valve becomes stationary and thus causes the steam-ports to reopen for the next stroke. By turning the stop K the lugs can be made to strike sooner or later, thus varying the expansion.

For the purpose of investigating the action of this gear the two valves are represented in their simplest form in Fig. 34 and are provided with a single steam-way for each end of the cylinder. To solve the problem stated above it is of no consequence whether there is one large steam-way or several smaller ones. The following investigation will show why it is that one wide steam-way is generally replaced by several narrow ones.

In Fig. 34 the axis X X passes through the centre of motion of the main valve S, and the latter is shown in its extreme left-hand position, the valve centre B is therefore at the distance of the eccentricity r from the centre of motion. While the lower valve moves to the left, the right-hand lug of the expansion-valve S_0 is kept back by the stop, in the position shown. The stop is so placed that its diameter in the direction of the valve-motion is $2x$; by turning the stop x can be varied from a certain minimum value x_1 to a certain maximum value x_2. The notation employed in the following discussion is inscribed in the figure and needs no special explanation.

If we now suppose the main valve S to be moved from its extreme left position to the right, the lug or shoulder will strike against the stop after both valves have traversed the distance y; the expansion-valve, and consequently its edge F_1, is therefore stationary; if now

the main valve continues its motion to the right, when it has moved through the distance $F_1 E_1 = s_1$ the steam-way will be closed and expansion will begin. The whole movement of the valve S to the right from its extreme position is therefore $y + s_1$; as it is now beyond the centre of motion and, according to the following discussion, must be beyond this centre, the valve movement ξ, or distance of the centre of the valve from the centre of motion, is

$$\xi = y + s_1 - r.$$

Now according to the figure,

$$L + r = s_1 + l + y + x,$$

which gives

$$\xi = L - l - x. \tag{130}$$

In the assumed valve-arrangement $L - l$ is constant, and this enables us, for every position x of the stop, to determine the position of the main valve which gives the crank position, the corresponding piston position, and the corresponding ratio of expansion. But all these determinations can be made with the help of the diagram. Before entering on these questions we will make the following remarks. The contraction of the left steam-way begins when the valve edge F_1 is at the right-hand edge of the port, i. e. when the valve S has travelled the distance $y + s_1 - a_0$ to the right from the extreme positions; in other words, when it is at the distance

$$\xi_1 = y + s_1 - a_0 - r,$$

from the centre of motion; according to preceding equations we also have

$$\xi_1 = L - l - x - a_0, \tag{131}$$

and for this valve movement we can, as before, readily find the corresponding crank position. In like manner, if we direct our attention to the right-hand side of Fig. 34, we can easily determine the position of the main valve at the instant when the *right-hand* steam-way is just beginning to open and when it is just closed. In the first case the valve has travelled from the extreme position a distance $y + s_2$, and in the second case the distance $y + s_2 + a_0$; or,

estimating the distances from the centre of motion, we have respectively the valve movements

$$\xi_2 = y + s_2 - r$$

and

$$\xi_3 = y + s_2 - r - a_0.$$

Now, according to the figure,

$$s_2 + L - r = l + x,$$

and, since $y = 2\,l_1 - 2\,x$, we have, by substituting the resulting values of y and s_2 in the foregoing equations, respectively:

$$\xi_2 = 2\,l_1 - (L - l) - x. \tag{132}$$

$$\xi_3 = 2\,l_1 - (L - l) - a_0 - x. \tag{133}$$

The four values for the valve movement given by equations (130) to (133) then correspond to crank and piston positions which are easily determined; but we will confine ourselves to the consideration of the valve movement ξ corresponding to the beginning of expansion; possibly the valve movement ξ_2 may be of interest, but it is easy to see that the reopening of the steam-way does not occur until after the piston has begun its return stroke, and this at once disposes of the only reason for a closer examination, the object of the latter being to ascertain whether steam is admitted twice during the same stroke. Of still less interest are the valve positions ξ_1 and ξ_3, the signification of which has already been explained.

We will now pass to the construction of the diagram, and will assume that the eccentricity r and angle of advance δ of the main valve's eccentric are given, and will construct the valve circle O D (Fig. 35, Pl. VI.). Measure the lengths L and l (Fig. 34) and describe, from O as a centre and O A = L − l as a radius, a second circle A R R_1, which we may also regard as the crank-pin circle; finally, if we lay off toward O the distance A X = x, we shall have O B = O A − A B = L − l − x = ξ = the valve movement given by equation (130); if, with O B as a radius, we describe a circle and continue it till it intersects the valve circle at P, O R will be the crank and L the piston position corresponding to the beginning of expansion, and this completes the presentation of the diagram for *Farcot's* gear, for it is easy to solve the inverse problem; namely that of determining

the position x of the stop for a given expansion. We simply determine the crank position O R corresponding to the piston position L; the distance R P measured on O R is the required value of x.

If we follow the intersection P on the valve circle we see that there is a minimum value for x, namely, $R_1 D_1 = x_1$; if x were taken smaller, the expansion-valve would not close the steam-way but simply narrow its opening. In this way we obtain an upper limit for expansion; at this limit the crank direction, at the instant of cut-off, coincides with the direction of the eccentricity $O D = r$; as δ is always greater than zero, piston position L_1 for greatest cut-off always falls in front of the middle point H of the stroke. In *Farcot's* gear, therefore, the ratio of expansion can never reach 0·5; this is a restriction which prevents *Farcot's* gear from being applicable to all cases.

On the other hand, with this gear the ratio of expansion may be reduced to zero. For if we make $x = A B_0$ (Fig. 35), this greatest value of x, which we will designate by x_2, will cause the steam to be cut off just at the beginning of the stroke. In designing a new gear we must assume the values of r, δ and the minimum value x_1; in our example we have taken $r = 40^{mm}$ (1·57″), $\delta = 15°$, and $x_1 = 15^{mm}$ (0·59″). Then, according to Fig. 35, we have:

$$L - l = r + x_1, \tag{134}$$

and for the maximum value $A B_0 = x_2$:

$$x_2 = L - l - r \sin \delta. \tag{135}$$

In our case $L - l = 55^{mm}$ (2·17″) and $x_2 = 44^{mm}$ (1·73″). There now remains the distance $2l$ to be determined (Fig. 34, Pl. VI.); in this determination we assume that $a_0 = s_1$, i.e. that the steam-way is fully open even when x is smallest.

But we then have:

$$a_0 + l + y + x_1 = L + r.$$

This gives, since

$$y = 2 l_1 - 2 x_1$$

and

$$r + x_1 = L - l,$$

the equation:

$$2 l_1 = 2 (L - l) - a_0. \tag{136}$$

But it is easy to see that we must also have:

$$2\,l_1 > 2\,x_2,$$

and this combined with equation (136) gives:

$$a_0 < 2\,(\mathrm{L} - l - x_2)\,.$$

In the present example, therefore, the width a_0 of the steam-way must be smaller than 22^{mm} (0·87″): if we employ two steam-ways, each 15^{mm} (0·59″) wide, the total width of the admission-port becomes $2\,a_0 = 30^{\mathrm{mm}}$ (1·18″), and, according to equation (136), we must make $2\,l_1 = 95^{\mathrm{mm}}$ (3·74″).

b. Gear with a Two-part Expansion-valve.

In the one-part valve-gear just discussed the opening and closing of the steam-ways is effected by the lugs striking against the stop; but this has the disadvantage, that the steam-ways in the expansion-valve must be considerably wider than those in the main valve. If, when x_1 has its minimum value, the distance s_1 between the edges F_1 and E_1 (Fig. 34, Pl. VI.) is equal to the width a_0 of the steam-way, as was assumed above, then the distance s_1 corresponding to the maximum value of x (which is x_2) is given by the equation:

$$s_1 + l + 2\,l_1 - x_2 = \mathrm{L} + r\,,$$

and making use of equations (136) and (134), we get:

$$s_1 = a_0 + x_2 - x_1\,;$$

the steam-way in the expansion-valve must at least have this width if it is to be fully open even when the expansion is greatest (i. e. when ratio of expansion is zero); now although this is not necessary, it is nevertheless true that with the one-part valve these steam-ways are wider than those in the main valve: it is on this account that two-part expansion-valves are preferred.

The main valve S (Fig. 36, Pl. VI.) carries the two gridiron-valves S_0 and S_0; each of these is provided on the inside with a lug n that strikes against the stop K; at the outer end of each plate there is a second lug m that strikes against a projection q.

Let us now examine the left half of the valve arrangement and

suppose the main valve to travel from left to right, the lug n will then strike the stop and the plate closes the steam-way at a certain piston position. Now, when the main valve returns, that is, moves from right to left, there will be a certain position in which the lug m strikes the projection and so places the expansion-valve that its steam-ways come over those of the main valve and thus again prepare for the admission of steam.

To investigate the action of this valve arrangement we have drawn the skeleton figure 37 on Pl. VI., and there inscribed the symbols that are to be used in what follows.

The main valve is shown in its extreme left-hand position, consequently the valve centre B is at the distance $BX = r$ from the centre of motion. In this position the lug m of the left valve-plate S_0 just touches the projection q, and one of the steam-ways a_0, which are here of the same width, stands over the other.

If we suppose the valve S to move from left to right, the expansion-valve will move with it the distance y and then stop. Then if the lower valve moves until it has travelled the additional distance a_0, cut-off will take place; the total travel of the main valve from the extreme position to the beginning of expansion is therefore $y + a_0$, or, expressed in terms of the valve movement ξ from the centre of motion, this becomes

$$\xi = a_0 + y - r;$$

but according to the figure:

$$L + r = a_0 + l + y + x,$$

consequently

$$\xi = L - l - x,$$

which is exactly the same equation that was obtained for the one-part valve; every part of the diagram given in Pl. VI., Fig. 35, is therefore applicable to the present valve-gear. We will add the following concerning the dimensions of the plates.

That the plate may shut the steam-way, even when x is at its minimum x_1, the edges F_2 must be to the right of E_2 (see Fig. 37, right-hand side), or at most the two edges may coincide; in the latter case we have

$$L - r = x_1 + l,$$

or exactly, as above,

$$L - l = r + x_1. \tag{137}$$

Finally the plate must have a certain prescribed length l_2. For if the stop is so set that the maximum value x_2 of x is in action, the left edge A of the expansion-plate must not open the steam-way, i. e. it must not pass to the right of the edge I. The extreme limit is when A coincides with I, in this case we have

$$l_2 = a_0 + s_2,$$

the distance between the edges F_2 and E_2 being again designated by s_2. Now according to the figure

$$L - r + s_2 = x_2 + l,$$

consequently

$$s_2 = r + x_2 - (L - l),$$

or, employing equation (137), we get

$$s_2 = x_2 - x_1.$$

From this follows the minimum length of the plate:

$$l_2 = a_0 + x_2 - x_1,$$

and this length may be easily determined from the diagram.

Recently attempts have been made to improve *Farcot's* gear so that it would give later cut-offs than are possible with the arrangement here discussed. At the Vienna Exposition there was an engine with a valve-gear in which the *Farcot* valve was not in immediate contact with the main valve but separated from it by a third valve driven by a special eccentric. (See *Dingler's* polyt. Journal, Jahrg., 1874, Article by *Müller-Melchiors*, p. 359.)

On the other hand, *Guhrauer* proposes (Zeitschrift des österreichischen Ingenieur- und Architekten-Vereines, 1872, Heft 16) to move the stops by means of an eccentric.

Although the object can be attained by these methods, it is open to question whether the increased work of friction does not more than neutralize the advantage obtained.

On the whole, *Georges'*, *Guinotte's* and *Meyer's* gears are preferable to *Farcot's* gear, even with the improvements last mentioned; and of the first three, *Meyer's* deserves the preference because it has the least valve-friction.

WORKS ON LINK-MOTIONS.

PHILIPPS, ingénieur des mines. Théorie de la Coulisse (de Stephenson) servant à produire la détente variable dans les machines à vapeur, et particulièrement dans les machines locomotives. Annales des Mines, Tome III. 1854.

Deutsch im Civilingenieur, Bd. I, S. 164. 1854. 'Theorie der variabeln Expansion mittelst STEPHENSON'S Coulisse.'

This treatise by *Philipps* is the first* one which was published on link-motions; it explains, but only in an analytical manner, *Stephenson's* valve-motion for equal and unequal angles of advance, as well as for open and crossed rods.

WEISBACH, Bergrath, Professor, Dr., Ingenieur- und Maschinen-Mechanik, Bd. III, S. 650,

Examines also *Stephenson's* valve-motion, but only in an analytical manner, and only for equal angles of advance and open rods. But *Weisbach* gives the general formula for the movement of the valve, somewhat different from the formula as given by *Philipps*, and was the first to give it in the form developed in the present treatise (Zeuner's).

ZECH, E., Ingenieur. Zeitschrift des österreichischen Ingenieur-Vereines. 1855. 7. Jahrgang.

Zech finds in his treatise, which was published almost simultaneously with that by *Weisbach*, the same formula for the movement of the valve in *Stephenson's* valve-motion, but gives also for the first time the entire theory of *Gooch's* valve-motion. Besides, *Zech* states at the end of his very elegant mathematical investigation, that the two equations which he obtains for the movements of the valves in both valve-motions, are the polar equations of circles, but he does not give any further attention to the matter.

* D. Kinnear Clark's 'Railway Machinery,' published in 1850, contained geometrical investigations of the action of the varieties of link-motions used on locomotives.—*Translator.*

REDTENBACHER, Hofrath, Professor. Gesetze des Locomotivbaues (Mannheim, 1855), S. 100: 'Die STEPHENSON'sche Taschensteuerung.'

Redtenbacher examines only *Stephenson's* valve-motion, but neglects several quantities in the approximate calculations to such an extent, that, although a simpler formula for the movement of the valve is obtained, the most remarkable peculiarity of *Stephenson's* valve-motion (variation of the lead) cannot be recognised in it. *Stephenson's* valve-motion, according to Redtenbacher, has a constant lead for all degrees of expansion.

After the treatises given above, the following ones were published by the author of the present book; all important valve-motions were examined, and the circle diagram applied for the first time.

Ueber die Coulissensteuerungen. Civilingenieur, Bd. II, S. 202. 1856.

Examines *Stephenson's* and *Gooch's* valve-motions for open and crossed rods.

Ueber die Dampfvertheilung bei den neuern Locomotivensteuerungen. Civilingenieur, Bd. III, S. 10. 1857.

Examines *Stephenson's* and *Hawthorn's* valve-motions; also those given by *Heusinger von Waldegg*, *Gonzenbach*, and *Meyer*.

Ueber die Diagrammsteuerung. Civilingenieur, Bd. III, S. 155. 1857.

Describes a simple model for use at lectures, by means of which the law of the motion of the valve of *all* link-motions may be shown in a clear manner.

Ueber die Locomotivsteuerung mittelst der STEPHENSON'schen Coulisse. Schweizerische polytechn. Zeitschrift, Bd. I. 1856.

REULEAUX, Professor. Die ALLAN'sche Coulissensteuerung. Civilingenieur, Bd. III, S. 92. 1857.

In this, as well as in all following treatises, the diagram is applied. Reuleaux gives here for the first time the entire theory of *Allan's* very ingenious valve-motion with a straight link.

LEHMANN, Ingenieur F. Die BORSIG'sche Locomotivsteuerung. Organ für die Fortschritte des Eisenbahnwesens, Bd. XIII, S. 241. 1858.

The treatise gives the description, theory, and the diagram of the double valve-motion, which is applied to most of *Borsig's* locomotives.

JENNI, Bergrath, Professor. Ueber die wichtigsten Constructionsverhältnisse und einige neue Anordnungen bei doppelt wirkenden stationären Hochdruckdampfmaschinen mit Schiebersteuerungen. Berg- und Hüttenmännisches Jahrbuch der K. K. Schemnitzer Bergacademie, Bd. VIII. Wien 1859.

This excellent treatise gives the entire theory of the simple valve-motion, as well as that by *Stephenson* and *Gooch*; it gives also, for the first time, the theory and diagram of *Pius Fink's* link-motion, without, however, considering the missing quantity.

VOLKMAR, Ingenieur, W. Eine neue Expansionssteuerung für Locomotiven. Civilingenieur, Bd. V, S. 179. 1859.

The treatise gives the entire theory of the double valve-motion invented by *Volkmar*, and which has been applied to engines of the North-Eastern Railway of Switzerland. This very ingenious mechanism is related to *Guinotte's* gear. *Volkmar* drives a *Polonceau* valve by a link of which one end is driven by an eccentric while the other is attached to the end of main valve rod. In a certain sense *Volkmar's* gear is the intermediate link that connects *Polonceau's* gear with *Guinotte's*, for in *Volkmar's* gear the relative, straight, line of centres is inclined to the axis of abscissas, but passes through the centre of crank-shaft.

FUHST, Ingenieur, H. 'Untersuchungen über die MEYER'sche variable Expansion' und 'Die MEYER'sche variable Expansion als Locomotivsteuerung.' DINGLER'S polytechn. Journal, Bd. 151, Heft 2 bis 5. 1859.

The first treatise contains a few special investigations of *Meyer's* valve-motion by means of the diagram; the author proposes in the other treatise to apply, instead of *Stephenson's* link, a link which swings round a fixed centre, and is moved by *one* eccentric.

KRAUSS, Maschinenmeister. Ueber Locomotivensteuerungen im Allgemeinen und insbesondere die Steuerung von POLONCEAU. Civilingenieur, Bd. VI, S. 110. 1860.

The author gives here, for the first time, a description of *Polonceau's* valve-motion, and examines its action by means of the diagram. Besides, the treatise compares the different valve-motions applied to locomotives, with respect to the distribution of the steam.

VON GRIMBURG, R., Ritter. Bemerkungen über die FINK'sche Steuerung. Zeitschrift des österr. Ingenieur-Vereins. 1862.

Contains the theory of the valve-motion (without considering the missing quantity), and proves by an example the coincidence of the results of the diagram, with those obtained by experiments on a model.

SCHMIDT, Gustav, Professor. Die Coulissensteuerungen. Zeitschrift des österr. Ingenieur-Vereins. 1866.

The treatise contains the investigation of the chief formulas, and of the diagrams of the valve-motions by *Stephenson*, *Gooch*, *Allan*, *Heusinger von Waldegg*, and *Fink*. *Schmidt* also determines here, for the first time, the missing quantity for *Fink's* valve-motion (a little different from the results of the present third edition of Zeuner's treatise on valve-motions), and the treatise gives for the other valve-motions a different way of determining the chief formulas, but besides these, it contains no results which are not to be found in the former editions of this book.

MÜLLER, Ober-Ingenieur, O. H. Ueber Umsteuerungen, besonders für Schiffsmaschinen. Zeitschrift des Vereins deutscher Ingenieure. Bd. X. 1866.

This article examines with great knowledge the reversing motions as applied to marine engines, and finally points out the advantages of *Fink's* valve-motion for this very class of engines.

MAW, W. H., and SLADE, FRED. J. Locomotive Engineering and the Mechanism of Railways. By ZERAH COLBURN. 1867.

This work contains several chapters, by W. H. Maw and F. J. Slade, devoted to the geometrical investigation of the various link-motions in general use on locomotive engines, including *Stephenson's*, *Gooch's*,

Allan's, and *Heusinger von Waldegg's*. The methods of adapting the valve-gears to the various circumstances met with in practical construction are also considered.

VOLKMAR, Ingenieur, W. Ueber Verbesserungen an den Expansionssteuerungen mit einem Schieber. Organ für die Fortschritte des Eisenbahnwesens. 1868.

An interesting article, which examines minutely the distribution of the steam of the valve-motions with one valve, and which shows the advantages of *Trick's* port-valve.

NAPIER AND MACQUORN RANKINE. On the use of movable seats for slide-valves. (The Engineer, October 1867. See also Deutsche Industrie-Zeitung, Jahrg. 1868.)

This article describes the ingenious valve arrangement discussed in the present edition under the head of "*Napier* and *Rankine's* Cut-off Gear."

PICHAULT, S., Ingénieur. "Diagrammagraphe." Procédés théoriques et pratiques pour établir et pour vérifier les distributions des machines à vapeur.

'Annales industrielles,' 1871. This has also been published as a separate paper: Paris, 1872. It describes a device for generating our polar diagram mechanically; as the diagram thus drawn is an exact one, it shows the disturbing influence of the length of the eccentric rods on the quantities given by the diagram. *Pichault* bases his work on my propositions and formulas, but incorrectly states, in this paper and in another mentioned below, that I was assisted by the French engineer *Philipps* in establishing my polar diagram. *Philipps* did write a paper which in its second part discusses the polar diagram, but it was not sent to the Academy of Sciences in Paris till 1860, and *Philipps* himself conceded at the time ('Comptes rendus,' 1860, p. 935) my undoubted priority in the matter, and that he could not contest it.

GUINOTTE, Lucien, Ingénieur. Étude générale sur la détente variable et spécialement sur son application aux machines d'extraction. Système nouveau applicable à toutes espèces de machines à vapeur. Mons, 1871.

This excellent treatise has already been discussed in this book and received due consideration and appreciation. A good outline of *Guinotte's* paper may be found in the Polytechnischen Centralblatt, 1871, p. 1393.

HUNAEUS, O., Ingenieur. Die Anwendung der geraden Coulisse bei der FINK'schen Umsteuerung. Civilingenieur, Bd. 19. 1873.

The contents of this article are stated on p. 167 of this book.

MÜLLER-MELCHIORS, Ingenieur. Die Dampfmaschinensteuerungen auf der Wiener Weltausstellung 1873. Dingler's polytechnisches Journal 1874, S. 1.

This article presents clearly and discusses thoroughly the valve-gears at the Vienna Exposition; some of the more important valve-gears are investigated with the aid of the diagram.

PICHAULT, S., Ingénieur. Étude sur les appareils de distribution de vapeur, exposés à Vienne en 1873. 'Annales industrielles,' 6. année. 1874.

A very comprehensive and, so far as the mathematical treatment is concerned, an elegantly written treatise, which gives special attention to *Guinotte's* gear, discussing the latter with great detail; we have quoted it in the text. When deducing the valve movement for *Gooch's* link-motion under the supposition that the eccentricities, angles of advance, &c., were unequal, I followed, at the beginning of the investigations, *Pichault's* line of thought, but not later on, for in order to obtain certain rules or propositions given by *Guinotte*, *Pichault* goes too far in the way of neglecting certain terms of the formulas, namely, without saying so, he finally assumes infinitely long eccentric rods.

DWELSHAUSEN-DERY. Exposition universelle. Revue des machines motives. Revue universelle des mines, etc., de Cuifer. 20. année. Tome XXXIV. 1873.

Among other things, the report treats with great detail the valve-gears at the Vienna Exposition. The report is one of the best works on the prime movers at the Exposition that have appeared.

BALGUERIE, A., Ingénieur. Note sur les réactions du coulisseau sur la coulisse de distribution et sur le retour brusque du levier de changement de marche dans le cas du renversement de la distribution. Bulletin de la Société d'Encouragement, etc., 1874, p. 73.

A highly interesting study concerning the intensity of the force with which the reversing lever of a link-motion tends to leave its position in

consequence of the resistance offered by the valve, and it also treats of the force to be exerted by the driver when reversing. The question is discussed analytically for the first time, and with the aid of the diagram, yielding interesting results. The question is also of practical importance, for the sudden flying back of the reversing lever that sometimes takes place is dangerous for the locomotive driver.

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Fig. 9.

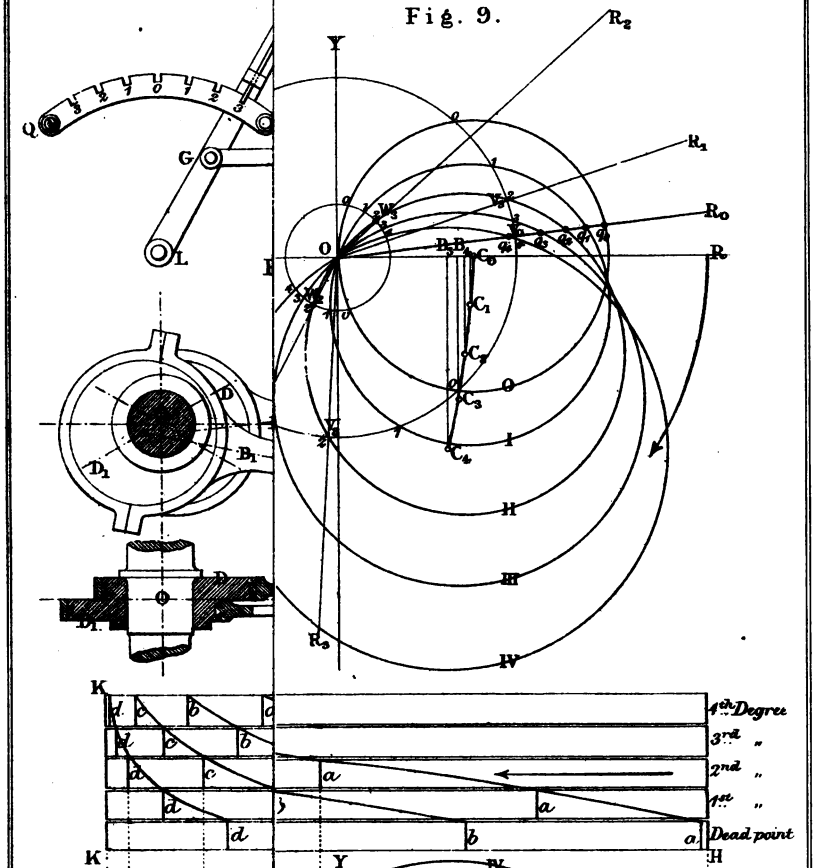

Fig. 10.

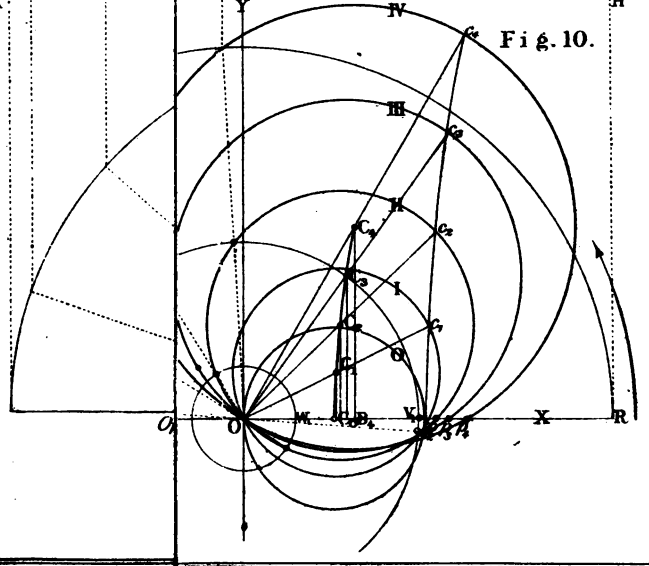

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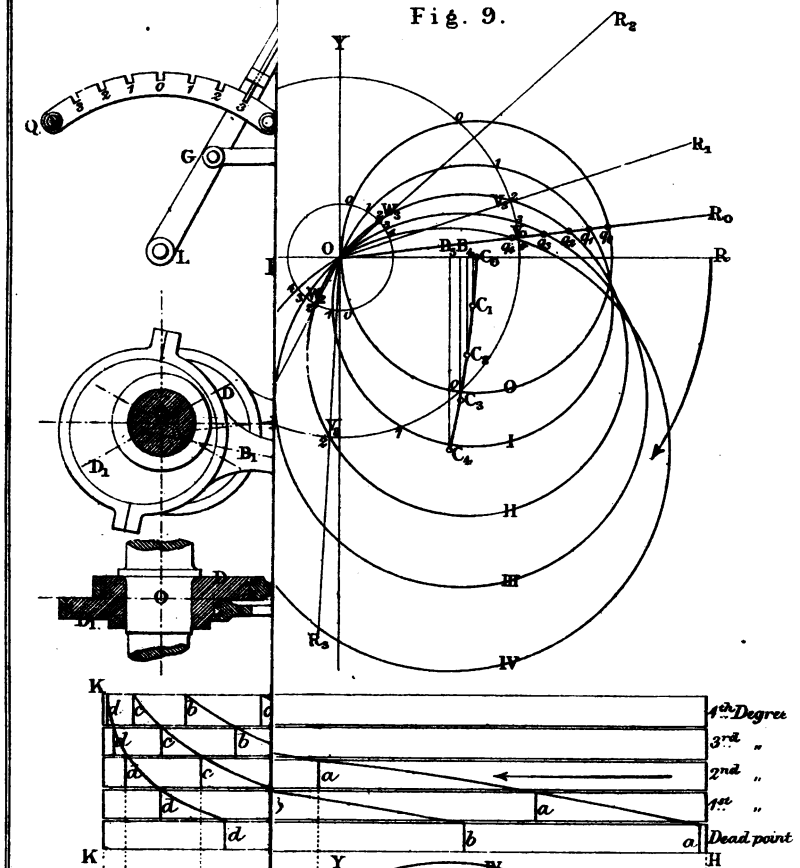
Fig. 9.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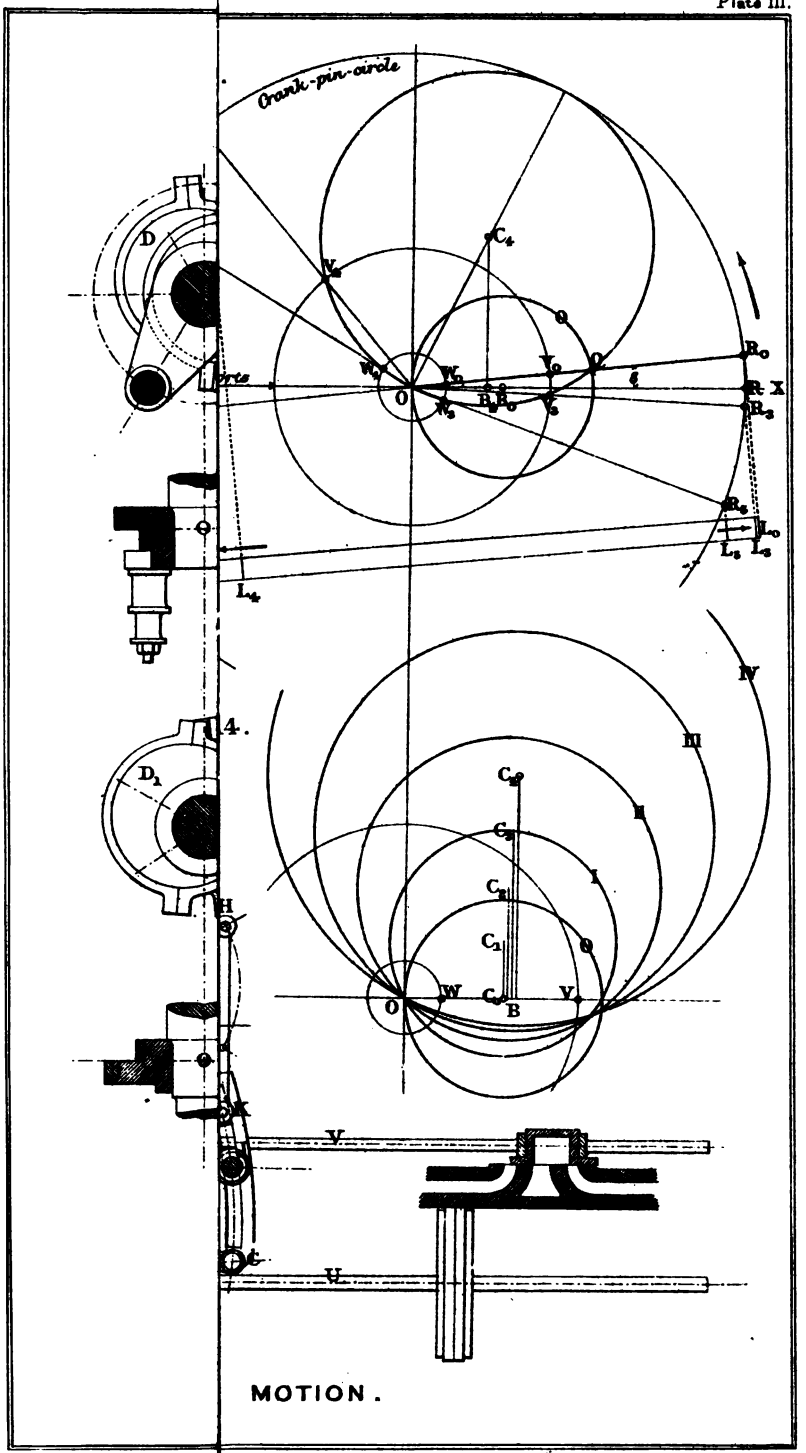

MOTION.

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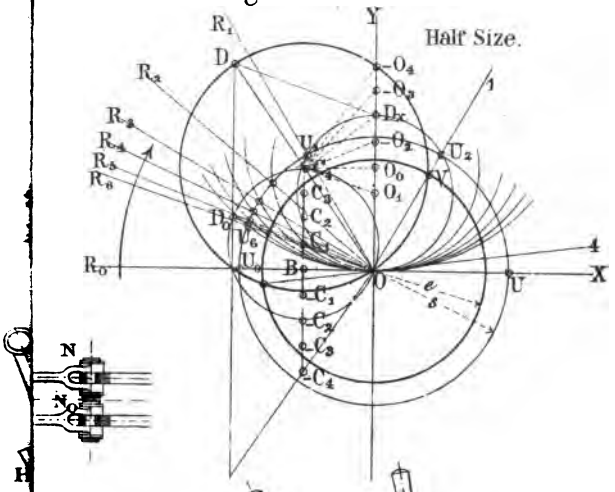
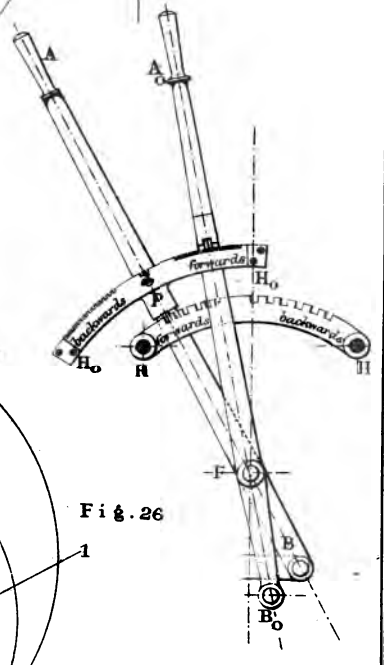
Fig. 27. Polonceau.





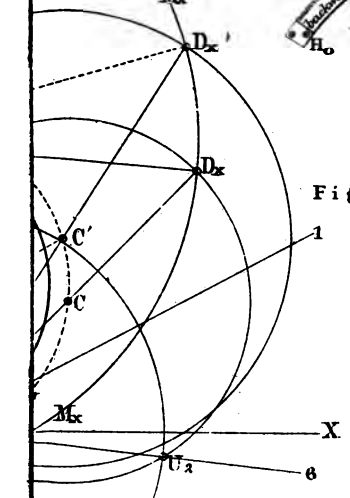


Fig. 26

Polonceau Meyer.

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Fig. 34.

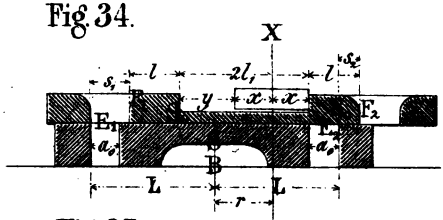


Fig. 37.

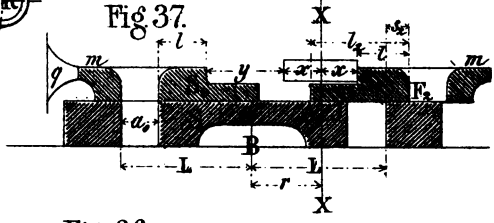


Fig. 36.

Farcot. (Two-port Valve.)

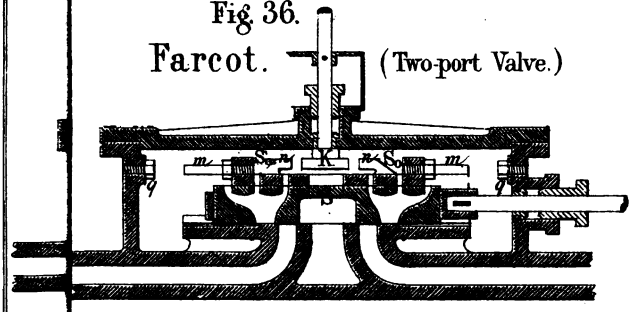
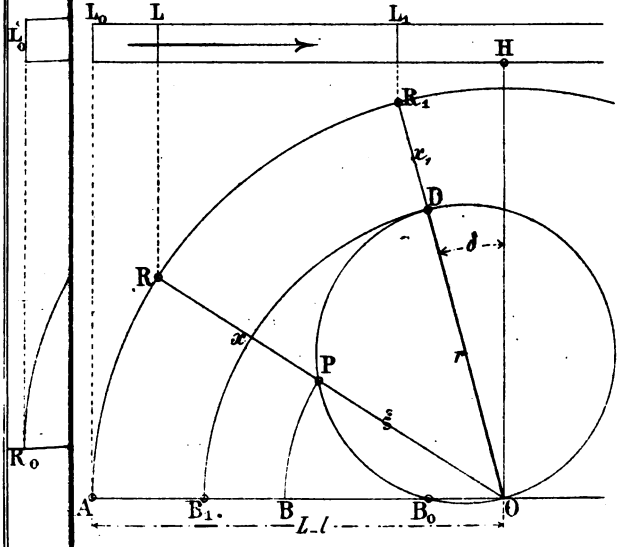


Fig. 35. Farcot.



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